Queries with Segments in Voronoi Diagrams

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Abstract

There are a number of proximity questions on sets of n points in the plane, such as Hausdorff distance between two point sets or nearest foreign neighbors for colored points, that can be answered in $O(n \log n)$ by computing Voronoi diagrams and querying with points. If we ask the same questions for line segments, we cannot use the same answer; although we can build the Voronoi diagrams for (disjoint) line segments, line segment queries may each intersect a linear number of cells.

In general, one would not expect segment queries to be efficient, since they could be used to solve "Hopcroft's problem"—determine if any of n given points lies on any of n given lines—and algorithms for Hopcroft's problem have an $\Omega(n^{4/3})$ lower bound in a natural model. We were surprised, therefore, that for n points and n disjoint segments, an algorithm can find the closest point to each segment in $O(n \log^3 n)$ time.

In particular, we show how to construct a data structure for a set P of n points that can report, in $O(\log n)$ time, the closest point to a query segment that is outside the convex hull $ch(P)$. The structure can be constructed in $O(n \log n)$ time, or in linear time if the Voronoi diagram of P is given. Then, for the n points P and a set of m disjoint segments S, not necessarily outside of the hull $ch(P)$, we can find the closest point to every segment $O((n+m)\log^3(n+m))$ time by using a divide-and-conquer algorithm that uses the previous data structure. This lets us compute Hausdorff distance and nearest foreign neighbors for disjoint segments in the same time.

This is joint work with Sergei Bespamyatnik, who will present our paper at SODA'99. A full version is at `http://www.cs.ubc.ca/spider/snoeyink/papers/for-nhbr.ps.gz`