Kinetic Collision Detection Between Two Simple Polygons

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The problem of *collision detection* between moving objects is fundamental to simulations of the physical world. In most simulation systems, objects are checked for penetration at fixed time intervals. In this context, one wishes to take advantage of the *temporal coherence* between time steps: some information computed at time t (e.g. a sorted list of coordinates) is used as a starting point to compute its updated value at time $t + \Delta t$ [5].

Our goal is to develop practical methods that do not require recomputations at fixed time intervals, yet detect collisions exactly when they happen. For this purpose, we take the point of view of *kinetic data structures* [1], and we focus here on collision detection between two polygons in rigid motion. What makes this problem challenging is that the two polygons can be quite intertwined and thus in close proximity in many places at once.

A kinetic data structure, or KDS for short, is built on the idea of maintaining a discrete attribute of objects in motion by animating a proof of its correctness through time. The proof consists of a set of elementary conditions, or *certificates*, based on the kinds of tests performed by ordinary geometric algorithms. Those of the certificates that can fail as a result of the rigid motion of the polygons are placed in an event queue, ordered according to their earliest failure time. When a certificate fails, the proof needs to be updated. Unless a collision has occurred, we perform this update and continue the simulation. The kinetic model allows us to perform a rigorous combinatorial time-cost analysis and obtain practical solutions at the same time.

For our collision detection problem, we keep track of a moving polygonal line separating the two polygon (the *relative convex hull* [7]). Such a polygonal line changes continuously, but its combinatorial description —a sequence of polygon vertices— changes only at discrete times. As its proof of correctness, we define a new structure called the *external relative geodesic triangulation* (ERGT). This structure is a planar map combining the ideas of the relative convex hull and of the *geodesic triangulation* of a simple polygon [2] (this latter structure was also used by Mount [4] for the static problem of intersection detection). In contrast to earlier hierarchical approaches that use rigid bounding volumes, the ERGT is made of a set of flexible shells surrounding each of the polygons (Figure 1). We show that it has the following properties:

1. The correctness of the ERGT is certified by a set of counter-clockwise certificates. It can be updated in logarithmic time when a certificate fails.

2. The number of certificates is only a logarithmic factor more than the the size of the minimum link separator for the two polygons [6]. Thus our separation proof automatically adapts to

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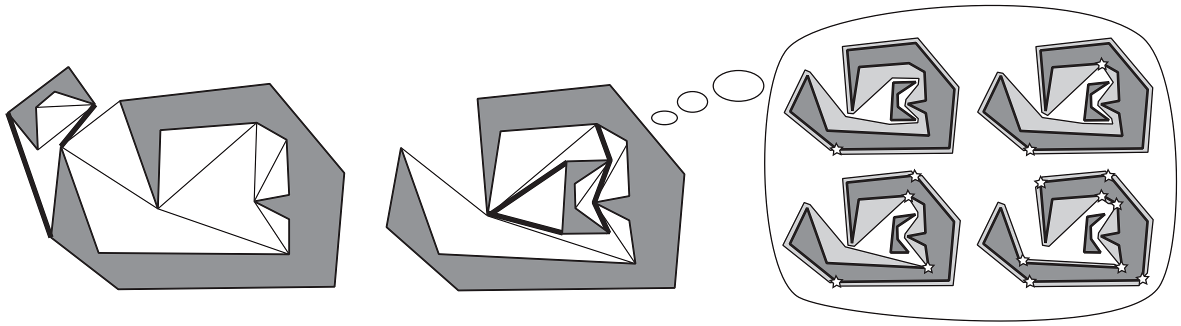

Figure 1: The ERGT is the planar map induced by a hierarchy of pinned rubber bands around each polygon, shown here for two distinct relative positions of two polygons. The right polygon is dreaming about the first four levels of its hierarchy.

the complexity of the relative placement of the two polygons—from a single separating line when the polygons are far apart to as complex as necessary when they have many points of near contact. This feature is important in the kinetic model, where objects are allowed to change their *motion plan* unpredictably, e.g. when they collide.

3. The number of certificate failures is roughly linear for translational motion, and roughly quadratic for rigid motion. These bounds hold assuming some usual algebraic or pseudo-algebraic conditions on the equations of motion, and should be compared with the naïve cubic bounds. The relative convex hull itelf can change roughly as many times as the ERGT, and the latter is therefore *efficient* in the sense of [1].

We believe that the kinetic setting is the framework of choice to approach collision detection problems, even when the motion plans are not fully known. In two dimensions, we would like to integrate the approach presented in this abstract with the distance sensitive approach for convex polygons of a companion paper [3], and to generalize the structure to the case of multiple moving polygons. The question remains of whether these ideas can be successfully adapted to three-dimensional non-convex bodies.

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