On Non Proximity-Drawbility of Maximal Planar Graphs [1]

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Abstract

It is known that some maximal planar graphs admit straight-line drawings of certain constrained triangulations and others do not admit such drawings, where these constrained triangulations include greedy triangulation, minimum weight triangulation, Delaunay triangulation, min-max edge length triangulation, Gabriel triangulation, relative neighborhood triangulation, general β triangulation, open (close) sphere triangulation, and closed rectangle triangulation. In this paper, we investigate the problem of what is the common criteria for maximal planar graphs which do not admit straight-line drawings of all these constrained triangulations. We provide several combinatorial necessary conditions for maximal planar graphs not admitting a straight-line drawing of the above mentioned triangulations. We also identify some classes of maximal planar graphs which admit drawings as minimum weight and greedy triangulations, respectively.

1 Introduction

By Fary theorem [10], every planar graph $G = (V, E)$ corresponds to an isomorphic straight-line planar graph $G' = (V', E')$ in the plane, where E' are a set of straight-line segments. That is, there is a perfect match between V and V' and between E and E'. Therefore, a maximal planar graph must admit a straight-line drawing of triangulation. However, when G' must satisfy some constraints, such a G' may not exist. Dillencourt presented a necessary condition for a planar graph admiting a straight-line drawing of Delaunay triangulation [6]. Hence, he also provided a non drawability condition. Lenhart and Liotta studied the minimum weight drawing for a maximal outerplanar graph [17]. They discovered a characteristic of the minimum weight triangulation of a regular polygon using the combinatorial properties of its dual trees. By using this characteristic, they devised a linear-time algorithm that receives a maximal outerplanar graph G as input and constructs a straight-line drawing of G as output such that the output drawing is a minimum weight triangulation of the set of points representing the

[1] This work is supported by NSERC grant OPG0041629.

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vertices of G. Recently, Lenhart and Liotta [18] demonstrated some examples of 'forbidden' graphs for Minimum weight drawing, and also proved that if any graph contains such forbidden subgraph, then it is non minimum weight drawable. Interestingly, they showed that some graphs admit Minimum weight drawings but not Delaunay drawings. For the maximum weight triangulation, it has been shown that for any tree whose internal node of degree k connects at least $k-2$ leaf-nodes admits a maximum weight drawing in the plane, and the drawing can be realized in linear-time [23]. Furthermore, any graph containing a 'nonflat cycle' does not admit a maximum weight convex drawing [23]. But, in the case of nonconvex drawing of maximal planar graph, the drawability is not known. An exellent survey of proximity drawability can be found in [4]. For other drawability problem, Monma and Suri showed that a tree with degree of more than six does not admit a straight-line drawing of minimum spanning tree [20]. Eades and Whitesides proved that the realization of Euclidean minimum spanning trees of degree six is NP-hard [7]. MinMax edge length triangulation was investigated by Edelsbrunner and Tan [8]. Garbriel and relative neighborhood graphs were investigated by Toussaint [22]. The negative drawability results for Gabriel graph, relative neighborhood graphs, and for for relatively closest graphs were proposed in [2, 18, 19].

In this paper, we investigated some common conditions for non drawability of maximal graphs as the above mentioned constrained triangulations. In Section 2, we present some definitions and basic lemmas. In Section 3, we show some combinatorial necessary conditions for non-drawability of maximal planar graphs. In Section 4, we show that maximal 'web' graph is minimum weight drawable, and caterpillar graph is greedy drawable. In Section 4, we conclude our work.