Finishing Lecture 18 – `MyLinkedList`

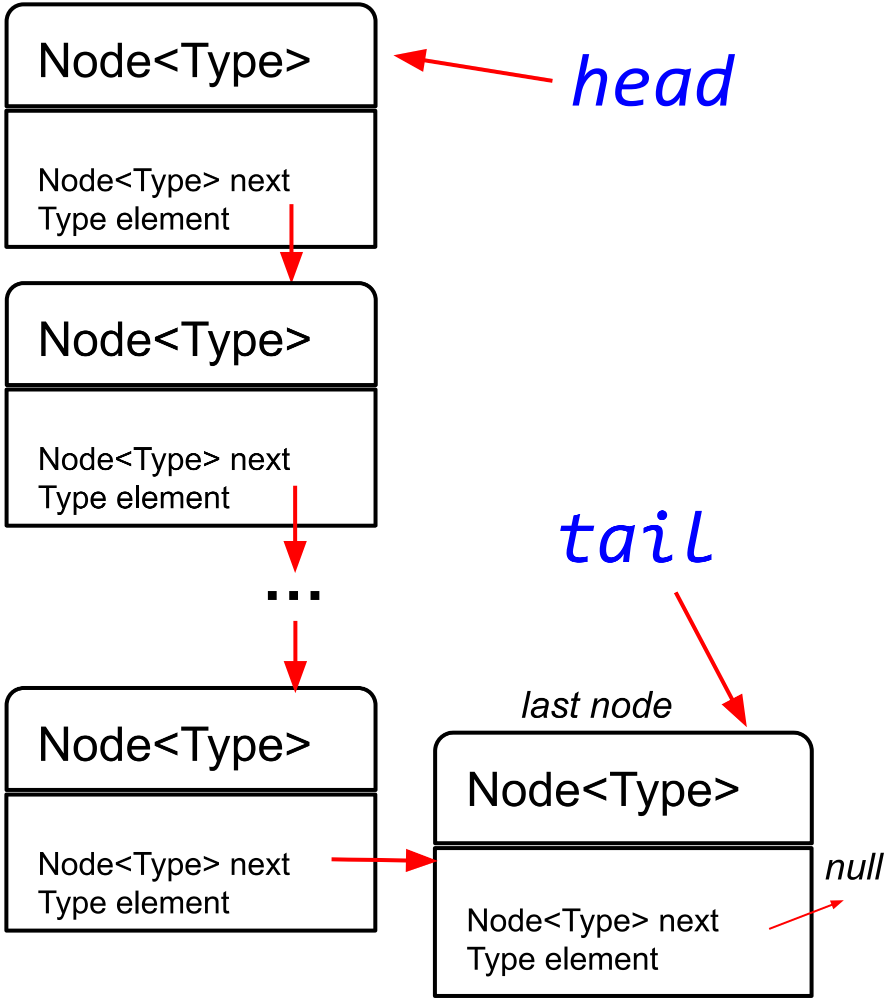
Review: Accessing Data Items

- Variables: named by identifiers
  - local variables
  - parameters
  - instance variables
- Indexed items: named by index
  - in `Arrays`
  - in `ArrayLists`
- Referenced items: “named” by pointer
  - `next` and `element` in `node`s

Review: Accessing Nodes Via Pointers

`this.head.getNext();`

- This does not get `next` field of `head`, which doesn't have such a field, being just a pointer
- Instead, read this as "get `next` field of the node `head` points to"
- What does `this.tail.getNext()` produce?
- What does `this.tail.getElement()` produce?
- note we can access a variable by its unique name, index, contents, or here, via a pointer



`remove` Method

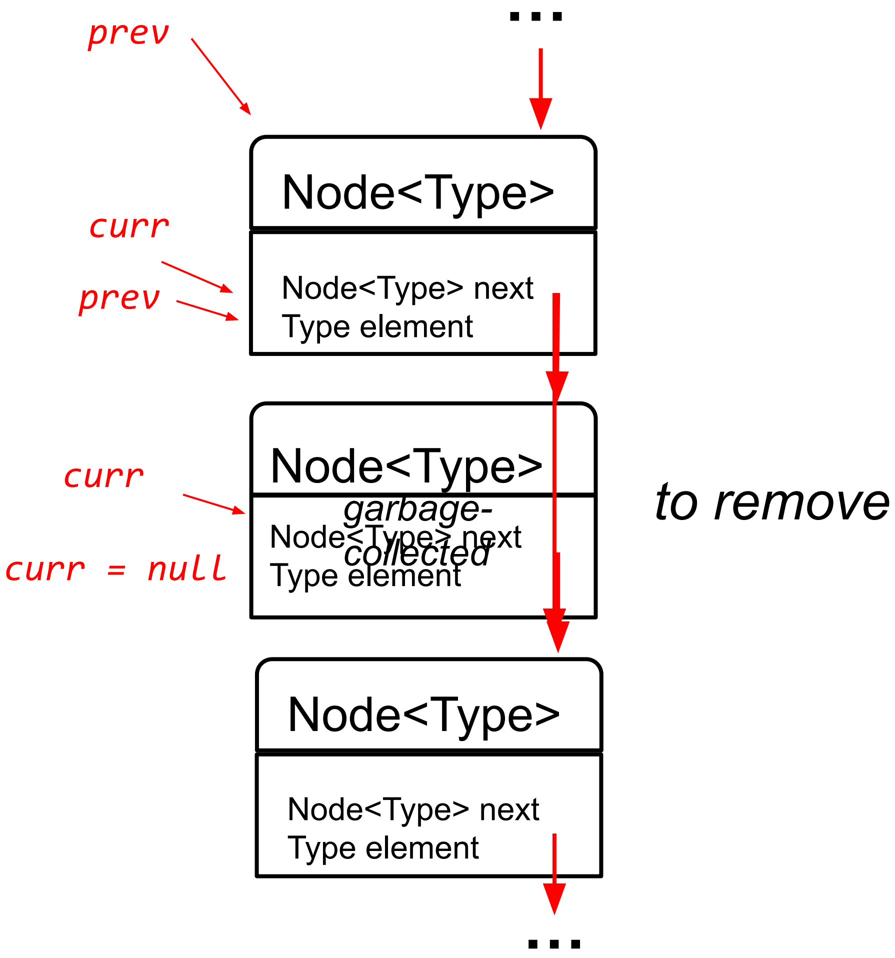
- We have implemented methods to remove first and last `element`s of `MyLinkedList`

- What if we want to remove *any* `element` from `MyLinkedList`?

- Let's write a general `remove` method
  - think of it in 2 phases:
    - a search loop to find correct element (or end of list)
    - breaking the chain to jump over the element to be removed

remove Method

- Search loop through Nodes until an element matches itemToRemove
- "Jump over" Node by re-linking predecessor of Node (using loop's prev pointer) to successor of Node (via its next reference)
- With no more reference to Node, it is garbage collected at termination of method



remove Method

- Edge Case(s)
  - again: can't delete from empty list
  - if removing first item or last item, delegate to `removeFirst`/`removeLast`
- General Case
  - iterate over list until `itemToRemove` is found in ptr-chasing loop
  - again: need `prev`, so we can re-link predecessor of `curr`. `Node` is GC'd upon `return`.

Note: caller of remove can find out if item was successfully found (and removed) by testing for `!= null`

```
public Node<Type> remove(Type itemToRemove){
    if (this.isEmpty()) {
        System.out.println("List is empty");
        return null;
    }
    if (itemToRemove.equals(this.head.getElement())) {
        return this.removeFirst();
    }
    if (itemToRemove.equals(this.tail.getElement())) {
        return this.removeLast();
    }
    //advance to 2nd item
    Node<Type> curr = this.head.getNext();
    Node<Type> prev = this.head;
    while (curr != null) {//pointer-chasing loop to find el.
        if (curr.getElement().equals(itemToRemove)) {
            prev.setNext(curr.getNext()); //jump over node
            this.size--; //decrement size
            return curr;
        }
        prev = curr;  //if not found, bop pointers
        curr = curr.getNext();
    }
    return null; //return null if itemToRemove is not found
}
```

`remove` Runtime

```
public Node<Type> remove(Type itemToRemove){
    if (this.isEmpty()) {                 // 1 op
        System.out.println("List is empty");     // 1 op
        return null;
    }
    if (itemToRemove.equals(this.head.getElement())) { // 1 op
        return this.removeFirst();          // O(1)
    }
    if (itemToRemove.equals(this.tail.getElement())) { // 1 op
        return this.removeLast();           // O(n) pointer chase till list end
    }
    Node<Type> curr = this.head.getNext();  // 1 op
    Node<Type> prev = this.head;          // 1 op
    while (curr != null) {                // n ops
        if (itemToRemove.equals(curr.getElement())) {  // 1 op
            prev.setNext(curr.getNext());        // 1 op
            this.size--;                  // 1 op
       return curr;          // 1 op
        }
        prev = curr;                      // 1 op
        curr = curr.getNext();                 // 1 op
    }
    return null;                          // 1 op
}
```

→ `remove(Type itemToRemove)` is O(n)

TopHat Question

Given that `animals` is a Singly Linked List of ***n*** animals, `curr` points to the node with an animal to be removed from the list, that `prev` points to `curr`'s predecessor, and that `curr` is not the tail of the list, what will this code fragment do?

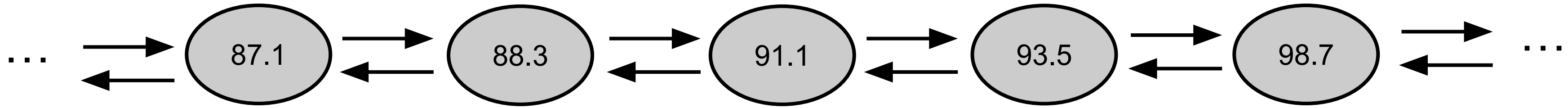
```
prev.setNext(curr.getNext());
curr = prev.getNext();
System.out.println(curr.getElement());
```

A. List is unchanged, prints out removed animal
B. List is unchanged, prints out the animal after the one that got removed
C. List loses an animal, prints out removed animal
D. List loses an animal, prints out the animal after the one that was removed

Doubly Linked List (1/3)

- Is there an easier/faster way to get to previous node while removing a node?
  - with Doubly Linked Lists, nodes have references both to next and previous nodes
  - can traverse list both backwards and forwards – Linked List still stores reference to front of list with `head` and back of list with `tail`
  - modify `Node` class to have *two* pointers: `next` and `prev`
  - eliminates pointer-chasing loop because `prev` points to predecessor of every `Node`, at cost of second pointer
    - classic space-time tradeoff!

Doubly Linked List (2/3)



- For Singly Linked List, processing typically goes from first to last node, e.g. `search`, finding place to insert or delete

- Sometimes, particularly for sorted list, need to go in the opposite direction
  - e.g., sort CS15 students on their final grades in ascending order. Find lowest numeric grade that will be recorded as an “A”. Then ask: who has a lower grade but is closer to the “A” cut-off, i.e., in the grey area, and therefore should be considered for “benefit of the doubt”?

Doubly Linked List (3/3)

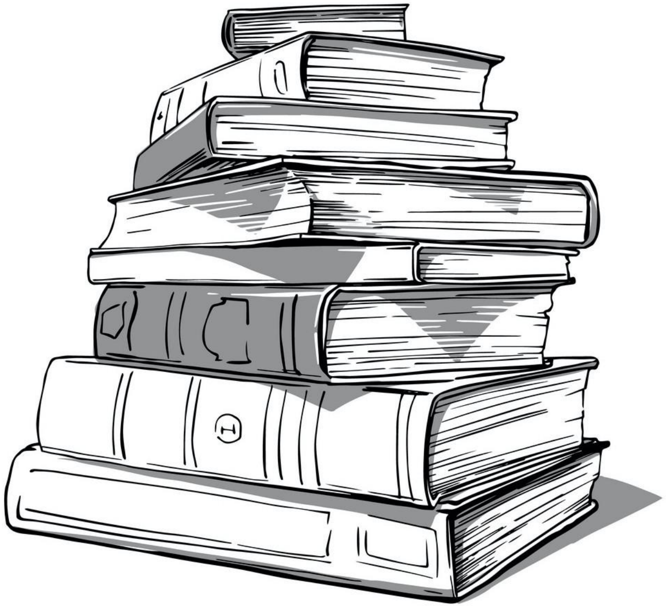
- This kind of backing-up can't easily be done with the Singly Linked List implementation we have so far
  - could build our own *specialized* `search` method, which would scan from the `head` and be, at a minimum, O(n)

- It is simpler for Doubly Linked Lists:
  - find student with lowest "A" using search
  - use `prev` pointer, which points to the predecessor of a node (O(1)), and back up until hit end of B+/A- grey area

Lecture 19

Stacks, Queues, and Trees

Stacks and Queues



Abstractions that are Wrappers for `MyLinkedList`

Outline

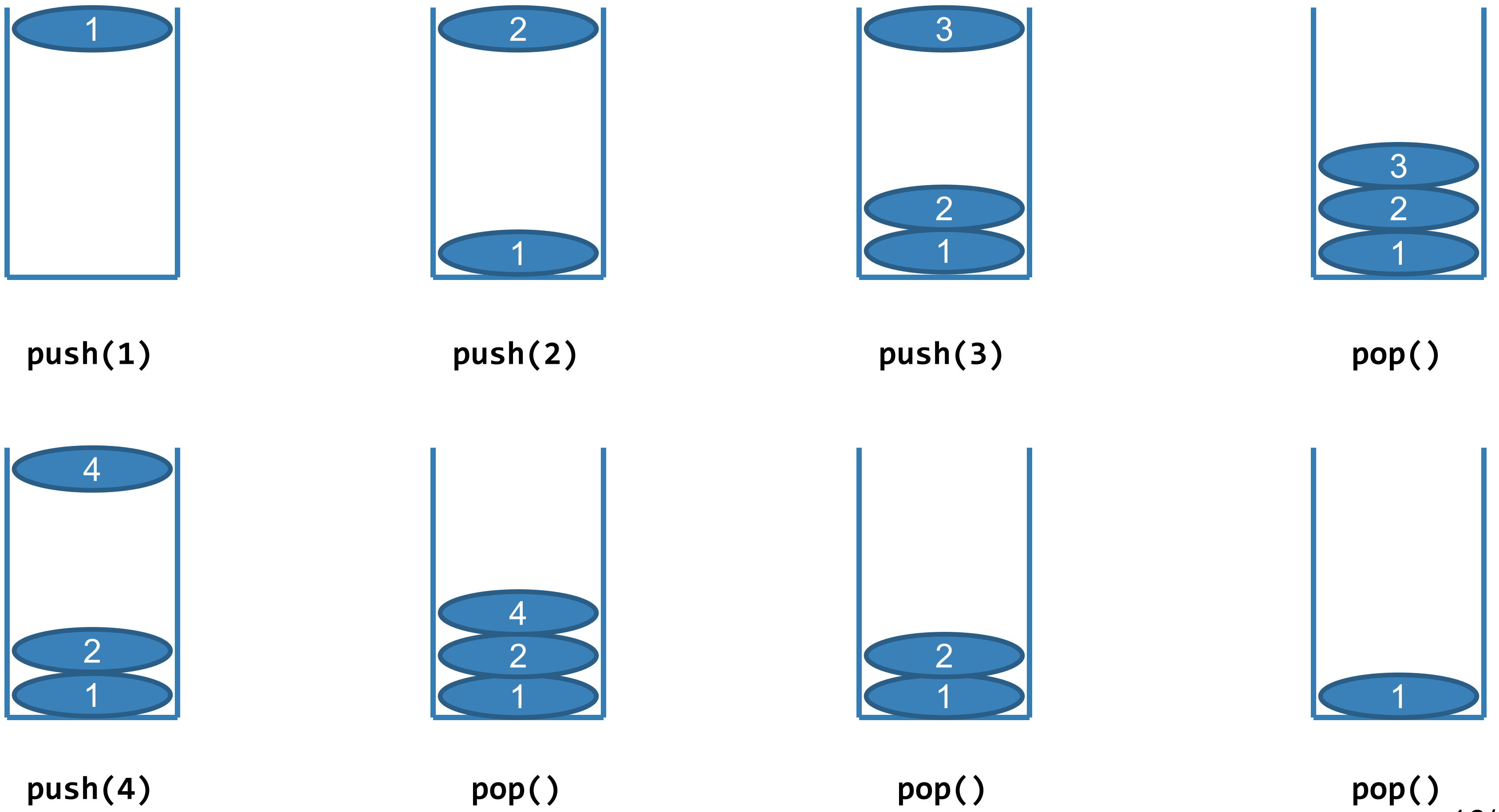
- Stacks and Queues
- Trees

Stacks

- `Stack` has special methods for insertion and deletion, and two others for size
  - `push` and `pop`
  - `isEmpty, size`
- Instead of being able to insert and delete nodes from anywhere in the list, can only add and delete nodes from top of `Stack`
  - **LIFO** (**L**ast **I**n, **F**irst **O**ut)
- We'll implement a stack with a linked list

Methods of a Stack

- Add element to top of `stack` — `public void push(Type el)`
- Remove element from top of `stack` — `public Type pop()`
- Returns whether `stack` has any elements — `public boolean isEmpty()`
- Returns number of elements in `stack` — `public int size()`


push(1)
push(2)
push(3)
pop()
push(4)
pop()
pop()
pop()

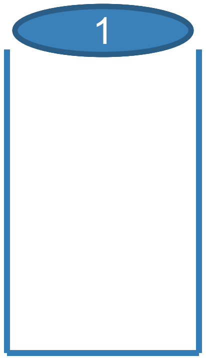
Stack Constructor

```
public class Stack<Type> {
    private MyLinkedList<Type> list;
    public Stack() {
     this.list = new MyLinkedList<Type>();
    }
    /* other methods elided */
}
```

- When generic `Stack` is instantiated, it contains an empty `MyLinkedList`
- When using a stack, you will replace `Type` with type of object your `Stack` will hold – enforces homogeneity
- Note: `Stack` uses classic “wrapper” pattern to modify functionality of the data structure, `MyLinkedList`, and to add other methods

Implementing Push

```
//in the Stack<Type> class ...
public Node<Type> push(Type newData) {
    return this.list.addFirst(newData);
}
```

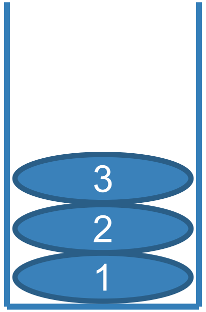


- Let’s see behavior...
- When element is `push`ed, it is always added to front of list
- Thus, `Stack` delegates to the `MyLinkedList`, `this.list` to implement `push`

Implementing Pop

- Let's see what this does...
- When popping element, it is always removed from top of `Stack`, so call `removeFirst` on `MyLinkedList` – again, delegation
- `removeFirst` returns element removed, and `Stack` in turn returns it
- Remember that `removeFirst` method of `MyLinkedList` first checks to see if list is empty

```
//in the Stack<Type> class ...
public Type pop() {
    return this.list.removeFirst();
}
```



`isEmpty`

- `Stack` will be empty if the `MyLinkedList, list,` is empty - delegation

- Returns `true` if `Stack` is empty; `false` otherwise

```
//in the Stack<Type> class ...
public boolean isEmpty() {
    return this.list.isEmpty();
}
```

size

- Size of `Stack` will be number of elements that the `MyLinkedList`, `list` contains – delegation

- Size is updated whenever `Node` is added to or deleted from `list` during `push` and `pop` methods

```
//in the Stack<Type> class ...
public int size() {
    return this.list.size();
}
```

TopHat Question

Look over the following code:

Who's left in the stack?

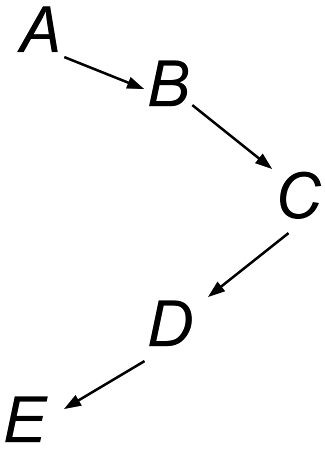
```
Stack<HeadTA> myStack = new Stack<>();
myStack.push(htaSarah);
myStack.push(htaAllie);
myStack.pop();
myStack.push(htaCannon);
myStack.pop();
```

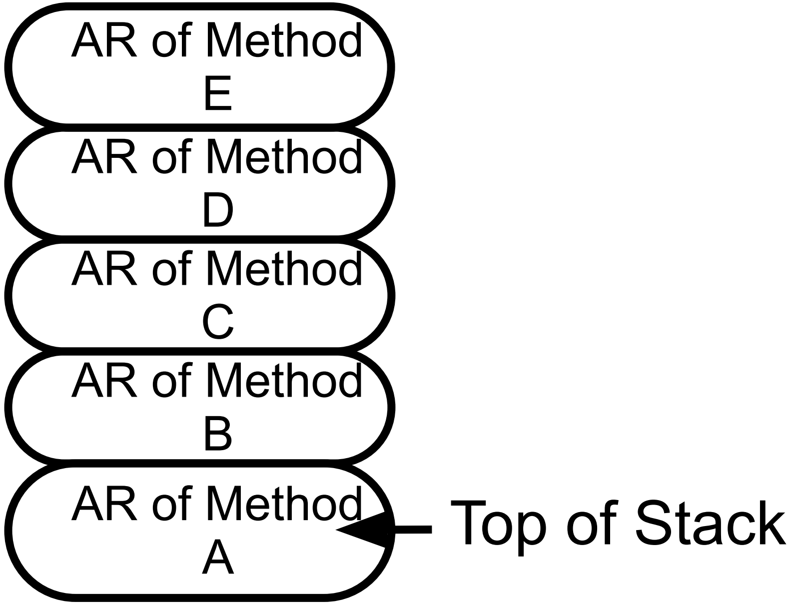
A. `htaSarah`
B. `htaAllie`
C. `htaCannon`
D. none of them!

Example: Execution Stacks

- Each method has an Activation Record (AR) – recall recursion lecture
  - contains execution pointer to next instruction in method
  - contains all local variables and parameters used by method
- When methods execute and call other methods, Java uses a `Stack` to keep track of the order of execution: "stack trace"
  - when a method calls another method, Java adds activation record of called method to `Stack`
  - when new method is finished, its AR is removed from `Stack`, and previous method is continued
  - method could be different or a recursively called clone, when execution pointer points into same immutable code, but different values for variables/parameters

Execution Stacks





A calls B
B calls C
… etc.

When E finishes, its AR is popped. Then D’s AR is popped, etc. Note this handles the tracking of invocations (clones) in recursion automatically

Stack Trace

- When an exception is thrown in a program, get a long list of methods and line numbers known as a stack trace

```
Exception in thread “main” java.lang.NullPointerException
   at DoodleJump.scroll(DoodleJump.java:94)
   at DoodleJump.updateGame(DoodleJump.java:44)
   ...
```

- A stack trace prints out all methods currently on execution stack

- If exception is thrown during execution of recursive method, prints all calls to recursive method

Bootstrapping Data Structures

- This implementation of the stack data structure uses a wrapper of a contained `MyLinkedList`, but user has no knowledge of that
- Could also implement it with an `Array` or `ArrayList`
  - `Array` implementation could be more difficult--`Array`'s have fixed size, so would have to copy our `Array` into a larger one as we push more objects onto the `Stack`
  - User's code should not be affected even if the implementation of `Stack` changes (true for methods as well, if their semantics isn't changed) – loose coupling!
- We'll use the same technique to implement a `Queue`

What are Queues?

- Similar to stacks, but elements are removed in different order
    - information retrieved in the same order it was stored
    - **FIFO: First In, First Out** (as opposed to stacks, which are **LIFO: Last In, First Out**)
- Examples:
    - standing in line for merch at the Eras Tour
    - waitlist for TA hours after randomization

Server at Seattle restaurant reminding herself what order customers get served in

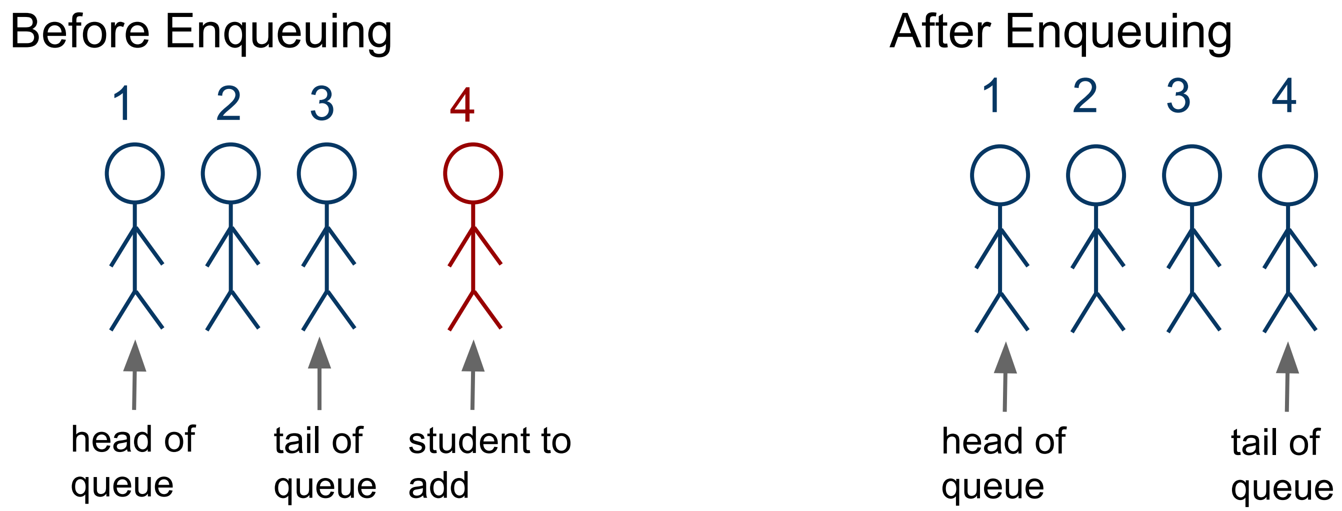
Methods of a Queue

- Add element to end of queue
- Remove element from beginning of queue
- Returns whether queue has any elements
- Returns number of elements in queue

```
public void enqueue(Type el)

public Type dequeue()

public boolean isEmpty()

public int size()
```

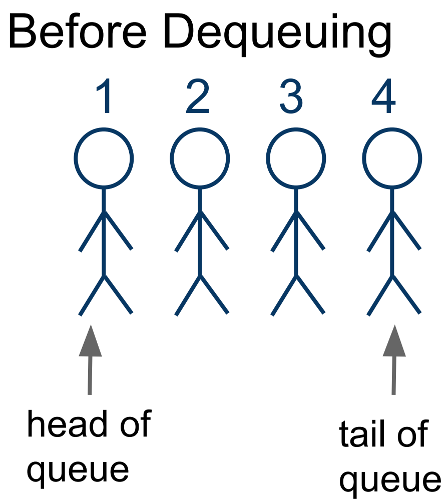
Enqueuing and Dequeuing

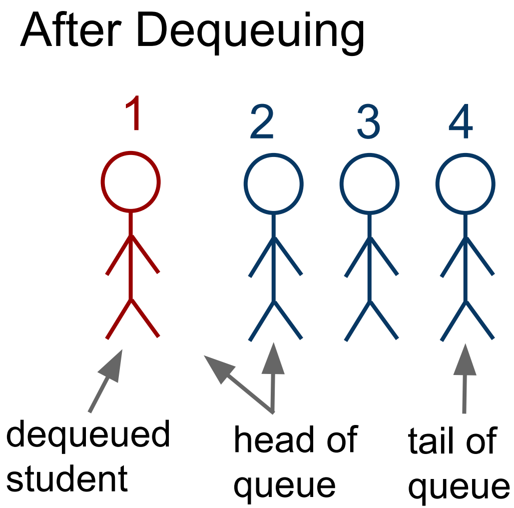
- Enqueuing: adds a node
- Dequeuing: removes a node



Enqueuing and Dequeuing

- Enqueuing: adds a node to the back
- Dequeuing: removes a node from the front





Our Queue

- Again use a wrapper for a contained MyLinkedList. As with Stack, we'll hide most of MLL's functionality and provide special methods that delegate the actual work to the MLL

```
public class Queue<Type> {

    private MyLinkedList<Type> list;

    public Queue() {
        this.list = new MyLinkedList<Type>();
    }
    // Other methods elided
}
```

- Contain a MyLinkedList within Queue class
  - enqueue will add to the end of MyLinkedList
  - dequeue will remove the first element in MyLinkedList

enqueue

- Just call `list`'s `addLast` method – delegation

```
public void enqueue(Type newNode) {
    this.list.addLast(newNode);
}
```

- This will add `newNode` to end of list

dequeue

- We want first node in `list`
- Use `list`'s `removeFirst` method – delegation

```
public Type dequeue() {
    return this.list.removeFirst();
}
```

- What if `list` is empty? There will be nothing to dequeue!
- Our `MyLinkedList` class's `removeFirst()` method returns `null` in this case, so `dequeue` does as well

`isEmpty()` and `size()`

- As with `Stack`s, very simple methods; just delegate to our wrapped `MyLinkedList`

```
public int size() {
   return this.list.size();
}

public boolean isEmpty() {
   return this.list.isEmpty();
}
```

TopHat Question

In order from head to tail, a queue contains the following: `katniss`, `gale`, `finnick`, `beetee`. We remove each person from the queue by calling `dequeue()` and then immediately `push()` each dequeued person onto a stack.

At the end of the process, what is the order of the stack from top to bottom?

A. `katniss, gale, finnick, beetee`
B. `katniss, beetee, gale, finnick`
C. `beetee, finnick, gale, katniss`
D. It's random every time.

Outline

- Stacks and Queues
- Trees

Trees

Searching in a Linked List (1/2)

- Searching for element in `LinkedList` involves pointer chasing and checking consecutive `Node`s to find it (or not)
  - it is sequential access
  - O(N) – can stop sooner for element not found if list is sorted

- Getting Nth element in an `Array` or `ArrayList` by index is random access (which means O(1)), but (content-based) searching for particular element, even with index, remains sequential O(N)

- Even though `LinkedLists` support indexing (dictated by Java's `List` interface), getting the `i`th element is also done (under the hood) by pointer chasing and hence is O(N)

Searching in a Linked List (2/2)

- For N elements, search time is O(N)
  - unsorted: sequentially check every node in list until element (“search key”) being searched for is found, or end of list is reached
    - if in list, for a uniform distribution of keys, average search time for a random element is N/2
    - if not in list, it is N
  - sorted: average* search time is N/2 if found, N/2 if not found (the win!)
  - we ignore issue of duplicates
- No efficient way to access N^{th} node in list (via index)
- Insert and remove similarly have average search time of N/2 to find the right place

**Actually more complicated than this – depends on distribution of keys*

Searching, Inserting, Removing

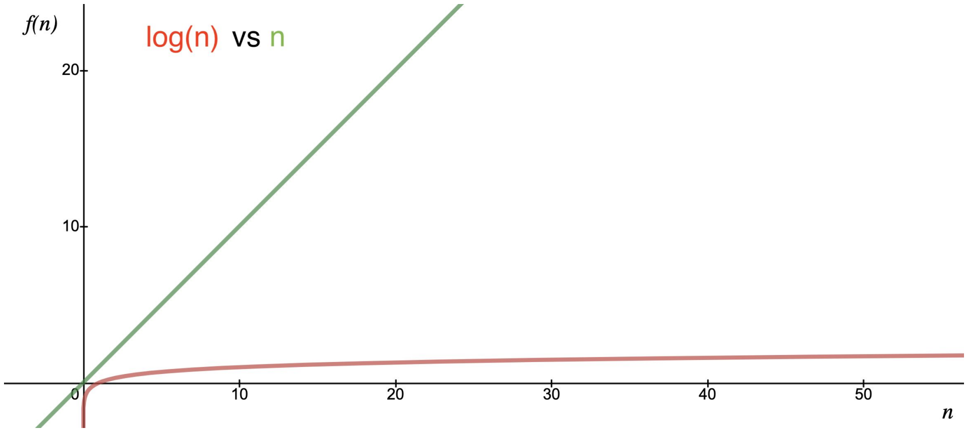
	Search if unsorted	Search if sorted	Insert/remove after search
Linked list	O(N)	O(N)	O(1)
Array	O(N)	O(log N) [coming next]	O(N)

Binary Search (1/4)

- Searching sorted linked list is sequential access
- We can do better with a sorted array that allows random access at any index to improve sequential search
- Remember merge sort with search $O(\log_2 N)$ where we did “bisection” on the array at each pass
- If we had a sorted array, we could do the same thing
  - start in the middle
  - keep bisecting array, deciding which portion of the sub-array the search key lies in, until we find that key or can’t subdivide further (not in array)
  - For N elements, search time is $O(\log_2 N)$ (since we reduce number of elements to search by half each time), very efficient!

Binary Search (2/4)

- $\log_2 N$ grows much more slowly than N, especially for large N



**relatively small n in this graph, but imagine how large the difference is as n increases*

N	(int) log(N)
1	0
10	3
100	7
1,000	10
10,000	13
1,000,000	17
10,000,000	20
100,000,000	23
1,000,000,000	27

Binary Search (3/4)

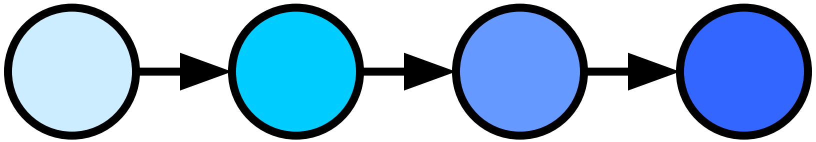
- A sorted `array` can be searched quickly using bisection because arrays are indexed
- `ArrayLists` (implemented in Java using arrays) are indexed too, so a sorted `ArrayList` shares this advantage! But inserting and removing from `ArrayLists` is slow (except for insertion and removal at either end)!
  - Inserting into or deleting from an arbitrary index in `ArrayList` causes all successor elements shift over. Thus insertion and deletion have same worst-case run time O(N)
- Advantage of `linkedLists` is insert/remove by manipulating pointer chain is faster [O(1)] than shifting elements [O(N)], but search can't be done with bisection 😞, a real downside if search is done frequently

Binary Search (4/4)

- Is there a data structure that provides both search speed of sorted arrays and `ArrayLists` and insertion/deletion efficiency of linked lists?
- Yes, indeed! `Trees`! They provide much faster searching than linked lists and much faster insertions than arrays!

Trees vs Linked Lists (1/2)

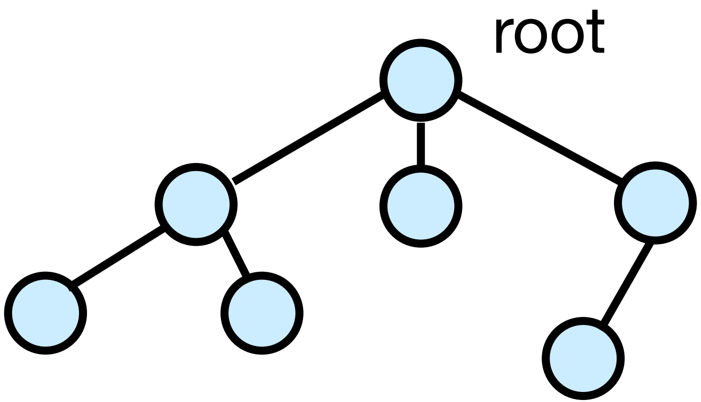
- Singly linked list – collection of nodes where each node references only one neighbor, the node's successor:



Trees vs Linked Lists (2/2)

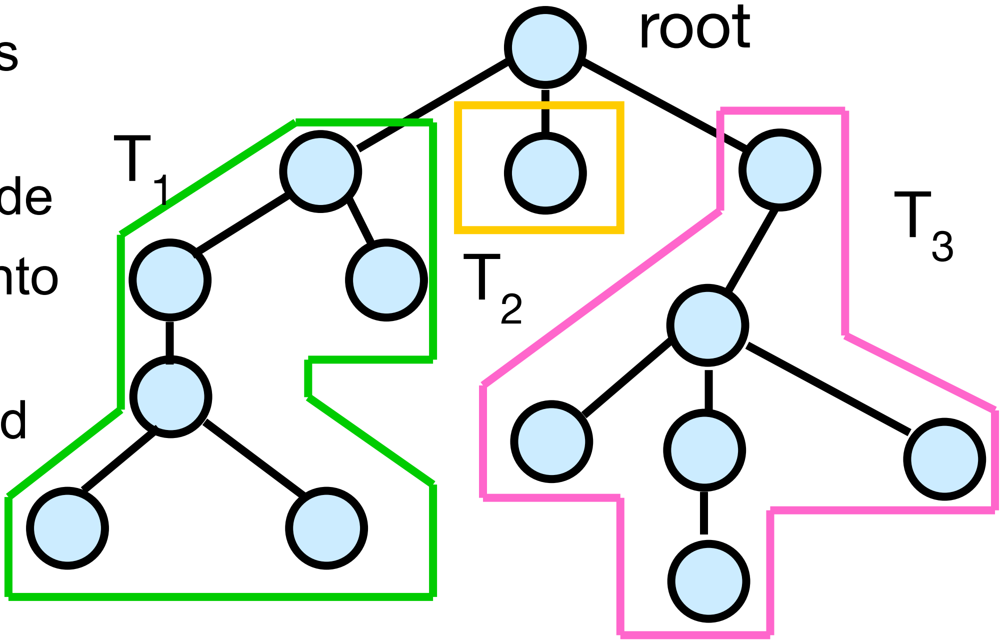
- Tree – also collection of nodes, but each node may reference multiple successors/children

- Trees can be used to model a hierarchical organization of data



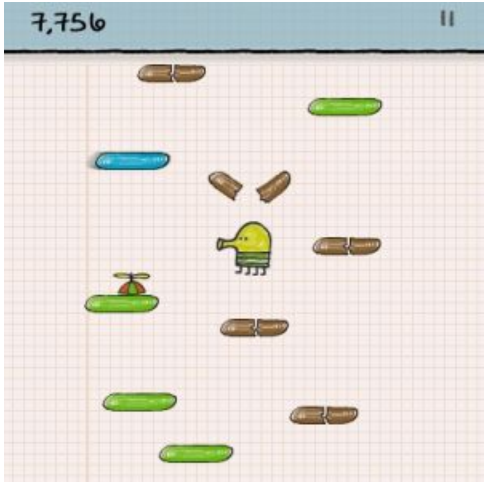
Technical Definition of a Tree

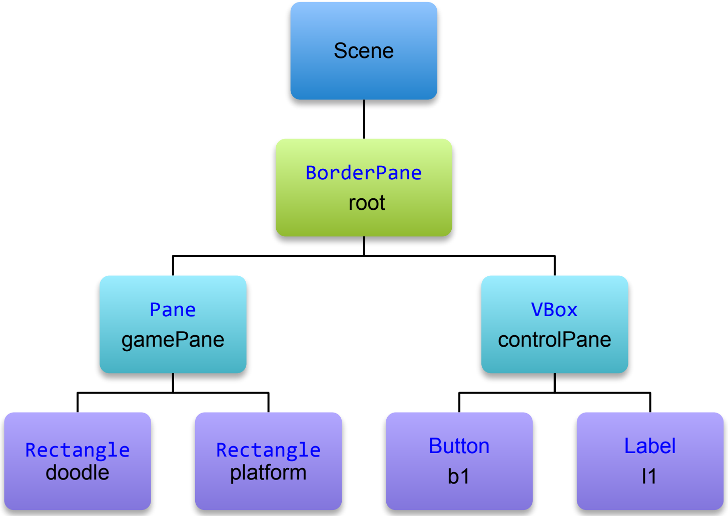
- Finite set, T, of one or more nodes such that:
  - T has one designated root node
  - remaining nodes partitioned into disjoint sets: $T_1, T_2, \ldots T_n$
  - each T_i is also a self-contained tree, called subtree of T
- Look at the image on the right- where have we seen seen such hierarchies like this before?



Graphical Containment Hierarchies as Trees

- Levels of containment of **GUI** components



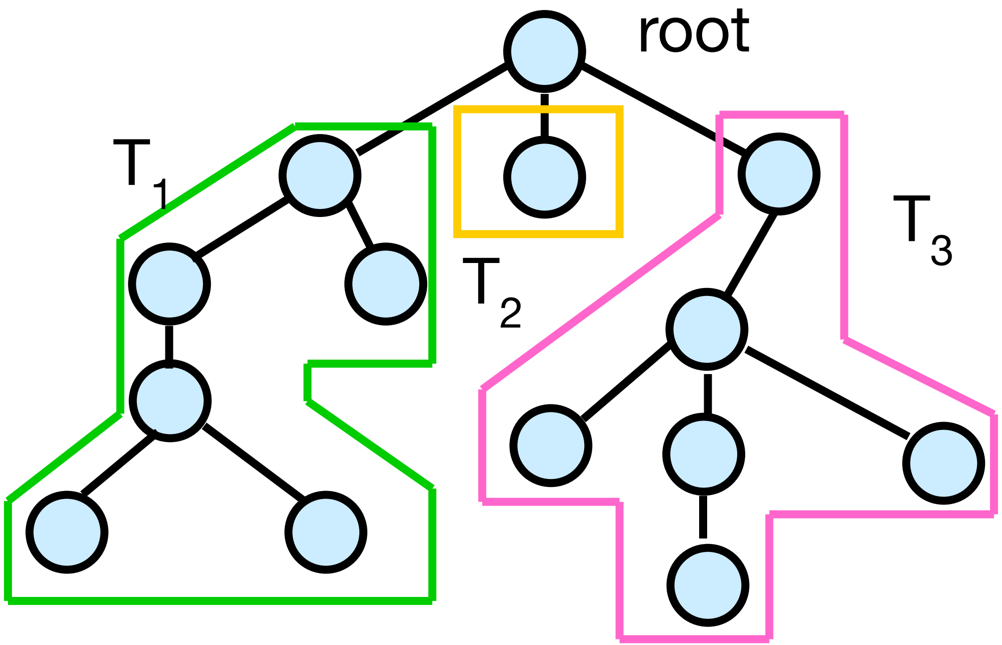


- Higher levels contain more components
- Lower levels contained by all above them
  - Panes contained by **root** pane, which is contained by Scene

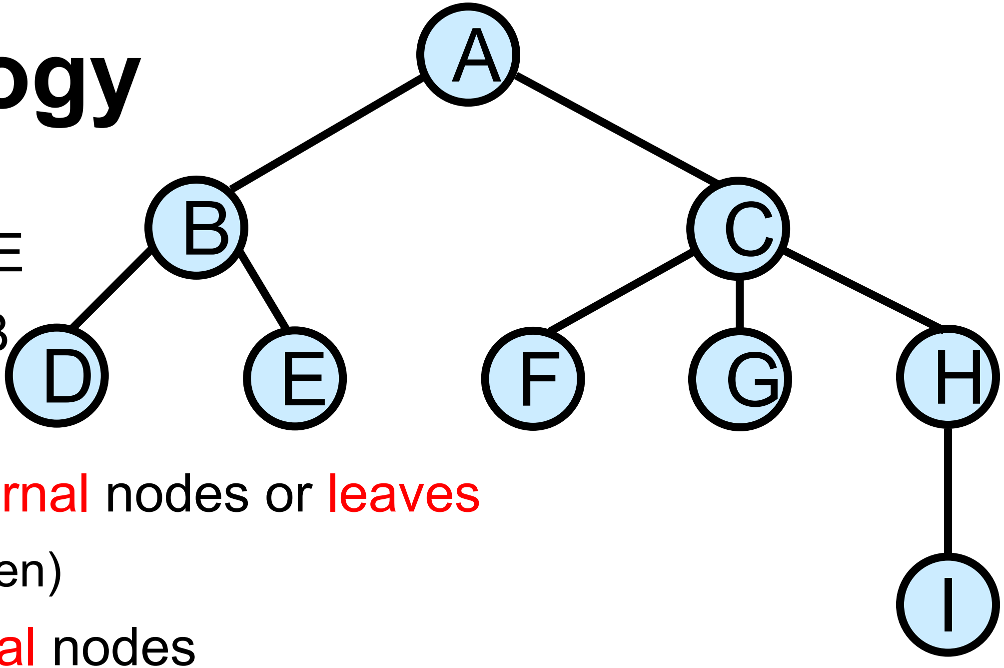
Tree Structure

- Note that the tree structure has meaning
  - any subtree of T, T_i, is also a tree with specific values

- Can be useful to only examine specific subtrees of T



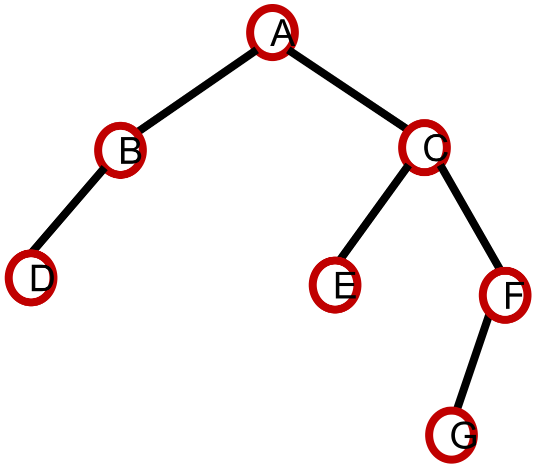
Tree Terminology



- A is the root node
- B is the parent of D and E
- D and E are children of B
- (C —— F) is an edge
- D, E, F, G, and I are external nodes or leaves
  - (i.e., nodes with no children)
- A, B, C, and H are internal nodes
- depth (level) of E is 2 (number of edges to root)
- height of the tree is 3 (max number of edges in path from root)
- degree of node B is 2 (number of children)

Binary Trees

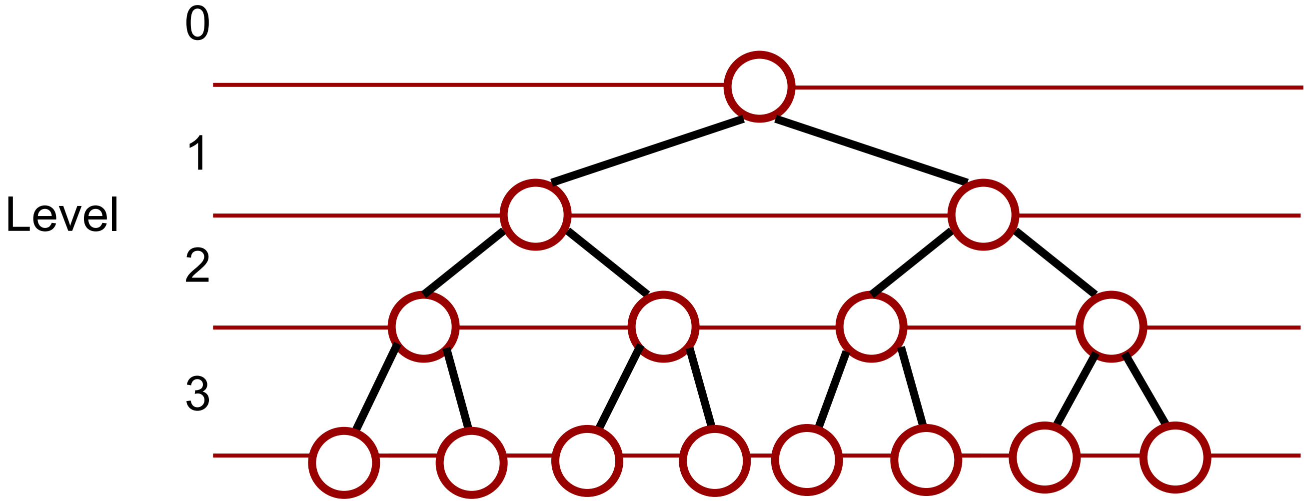
- Each internal node has a maximum of 2 successors, called children
  - i.e., each internal node has degree 2 at most
- Recursive definition of binary tree: A binary tree is either an:
  - external node (leaf), or
  - internal node (root) with one or two binary trees as children (left subtree, right subtree)
  - empty tree (represented by a null pointer)



- *Note:* These nodes are similar to the linked list nodes, with one data and two child pointers – we show the data element inside the circle

Properties of Binary Trees (1/2)

- A binary tree is full when each node has exactly zero or two children
- Binary tree is perfect when, for every level i, there are 2^i nodes (i.e., each level contains a complete set of nodes)
  - thus, adding anything to the tree would increase its height

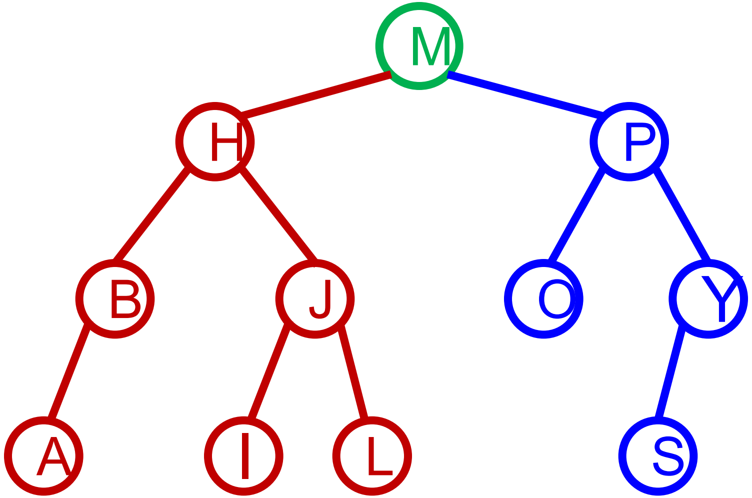


Properties of Binary Trees (2/2)

- *In a full Binary Tree:* (# leaf nodes) = (# internal nodes) + 1
- *In a perfect Binary Tree:* (# nodes at level i) = 2^i
- *In a perfect Binary Tree:* (# leaf nodes) <= $2^{(\text{height})}$
- *In a perfect Binary Tree:* (height) >= $\log_2$(# nodes) - 1

Binary Search Tree a.k.a BST (1/2)

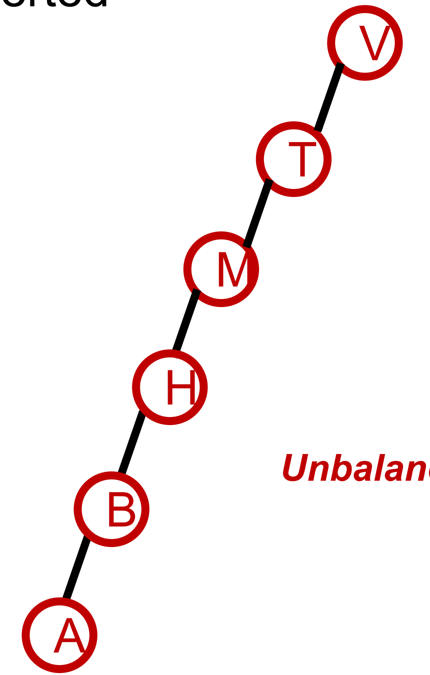
- Binary search tree stores keys in its nodes such that, for every node, keys in left subtree are smaller, and keys in right subtree are larger



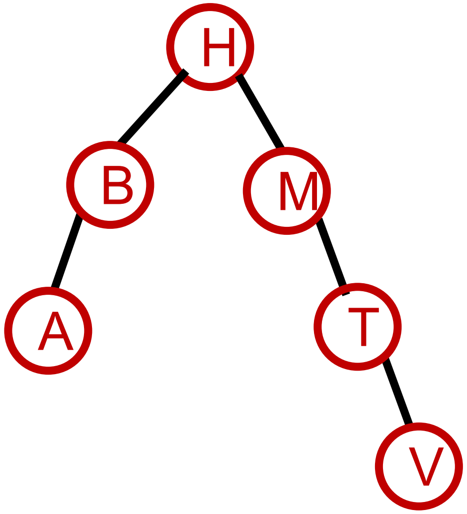
Note: the keys here are sorted alphabetically!

BST (2/2)

- Below is also BST but much less balanced. Gee, it looks like a linked list!
- The shape of the trees is determined by the order in which elements are inserted



Unbalanced



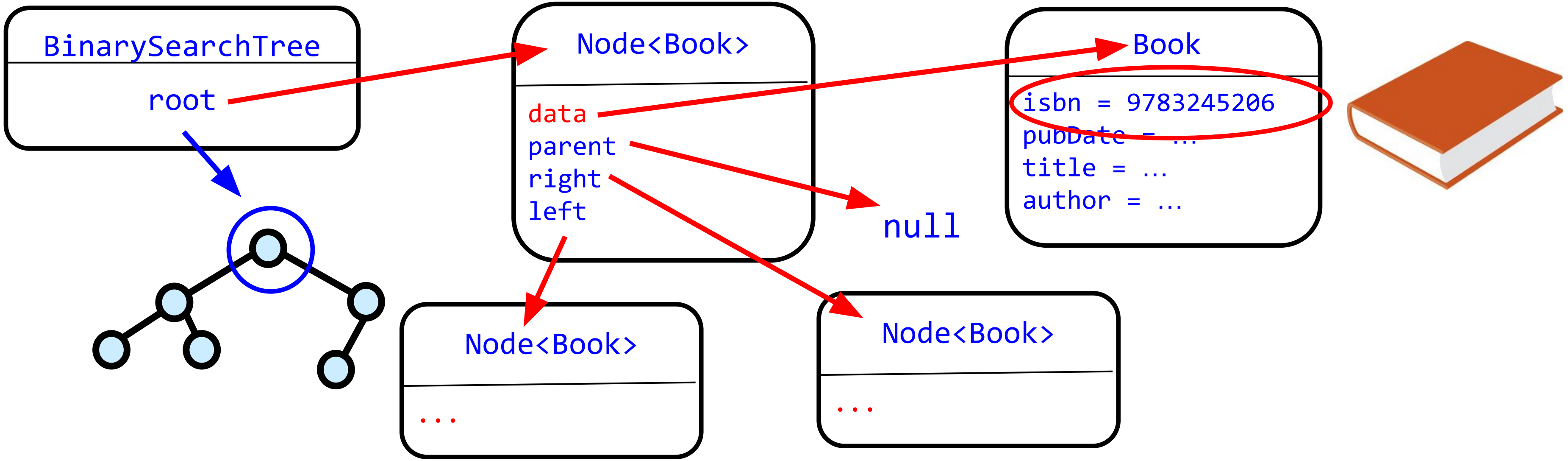
More Balanced

BST Class (1/4)

- What do BSTs know how to do?
  - much the same as sorted linked lists: *insert*, *remove*, *size, empty*
  - BSTs also have their own search method – a bit more complicated than simply iterating through its nodes

- What would an implementation of a BST class look like…
  - in addition to data, left, and right child pointers, we'll add a parent "back" pointer for ease of implementation (for the `remove` method – analogous to the `previous` pointer in doubly-linked lists!)
  - you'll learn more about implementing data structures in CS200!

Nodes, data, and keys



- data is a composite that can contain many properties,
- one of which is a key that Nodes are sorted by (here, ISBN #)

Comparable Book Class

- `compareBooks` is defined so we can easily compare the value of 2 books
  - returns number that is `<0`, `0` or `>0`, depending on the ISBN numbers
  - <0 if stored `this.isbn` < `toCompare`

```
public class Book {
    // variable declarations, e.g., isbn, elided
    public Book(String author, String title,
                      int isbn){
        //variable initializations elided
    }

    public int getISBN(){
        return this.isbn;
    }

    //other methods elided

    //compare isbn of book passed in to stored one
    public int compareBooks(Book compareBook){
        return (this.isbn - compareBook.getISBN());
    }
}
```

BST Class (2/4)

In our example, we use *Book* as *Type*

```
public class BinarySearchTree<Book> {

    private Node<Book> root;

    public BinarySearchTree(Book data) {
        //Root of the tree
        this.root = new Node(data, null);
    }

    // other methods shown next slide

}
```

- Our BinarySearchTree stores objects of type Book, meaning we will be able to use all methods Book has within our BST

If you'd like to see an example of a BST using a generic type that works for more than just books, check out slide 87 ;)

We'll go over what Node is in a few slides ☺

BST Class (3/4)

```
public class BinarySearchTree<Book> {

    private Node<Book> root;

    public BinarySearchTree(Book data) {
        //Root of the tree
        this.root = new Node(data, null);
    }

    public void insert(Book newData) {
      // . . .
     }
```

```
    //class continued
    public void remove(Book dataToRemove) {
       // . . .
    }

    public Node<Book> search(Book dataToFind) {
       // . . .
    }

    public int size() {
       // . . .
    }

} // end of class
```

BST Class (4/4)

- Our implementations of `LinkedLists`, `Stacks`, and `Queues` are "smart" data structures that chain "dumb" nodes together
  - the lists did all the work by maintaining previous and current pointers and did the operations to search for, insert, and remove information – thus, nodes were essentially data containers
- Now we will use a "dumb" tree with "smart" nodes that will delegate using recursion
  - tree will delegate action (such as searching, inserting, etc.) to its root, which will then delegate to its appropriate child, and so on
  - creates specialized `Node` class that stores its data, parent, and children, and can perform operations such as `insert` and `remove`

BST: Node Class (1/3)

- “Smart” `Node` includes the following methods:

```
// pass in entire data item, containing key and returns that item
public Node<Book> search(Book dataToFind);
// pass in entire data item, containing key and inserts into the tree
public Node<Book> insert(Book newData);
```

```
/* deletes Node pointing to dataToRemove, which contains key; removing Node
also will remove the matched data instance (here, a Book) unless there’s
another reference to it */
public Node<Type> remove(Book dataToRemove);
```

- Plus `set`ters and `get`ters of instance variables, defined in the next slides …

BST: Node Class (2/3)

- `Node`s have a maximum of two non-`null` children that hold data
  - four instance variables: `data`, `parent`, `left`, and `right`, with each having a `get` and `set` method.
  - `data` represents the data that `Node` stores. It also contains the key attribute that `Node`s are sorted by – we'll make a `Tree` that stores `Book`s
  - `parent` represents the direct parent (another `Node`) of `Node`–only used in `remove` method
  - `left` represents `Node`'s left child and contains a subtree, all of whose data is less than `Node`'s data
  - `right` represents `Node`'s right child and contains a subtree, all of whose data is greater than `Node`'s data
  - arbitrarily select which child should contain data equal to `Node`'s data

BST: Node Class (3/3)

```
public class Node<Book> {
    private Book data;
    private Book parent;
    private Node<Book> left;
    private Node<Book> right;
    public Node(Book data, Node<Book> parent){ //construct a leaf node as default
        this.data = data;
        this.parent = parent;
        //child ptrs null for leaf nodes; set for internal nodes when child is created
        this.left = null;
        this.right = null;
    }
   // will define other methods in next slides…
}
```

Smart Node Approach

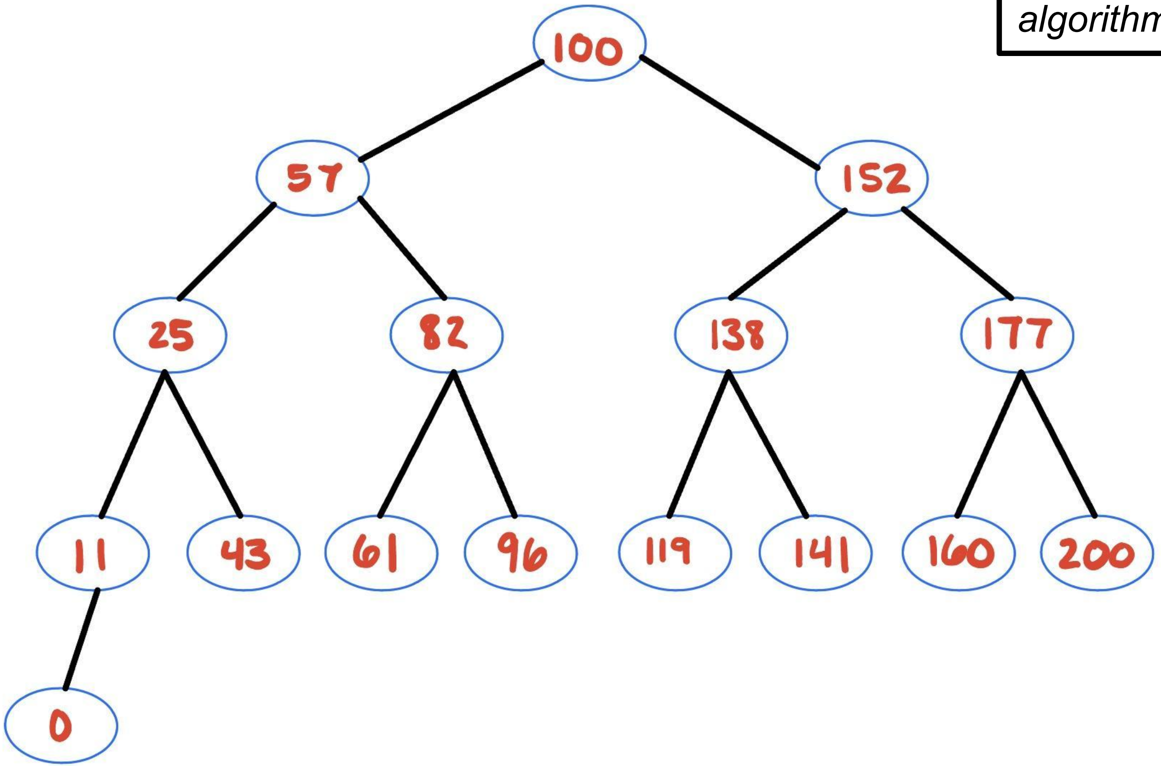
- BinarySearchTree is "dumb," so it delegates to root, which in turn will delegate recursively to its left or right child, as appropriate

```
// search method for entire BinarySearchTree:
public Node<Book> search(dataToFind) {
        return this.root.search(dataToFind);
}
```

- Smart node approach makes our code clean, simple and elegant
  - non-recursive method is much messier, involving explicit bookkeeping of which node in the tree we are currently processing
    - we used the non-recursive method for sorted linked lists, but trees are more complicated, and recursion is easier – a tree is composed of subtrees!

Let’s Search a BST

For a step-by-step walkthrough of this algorithm, see slide 100



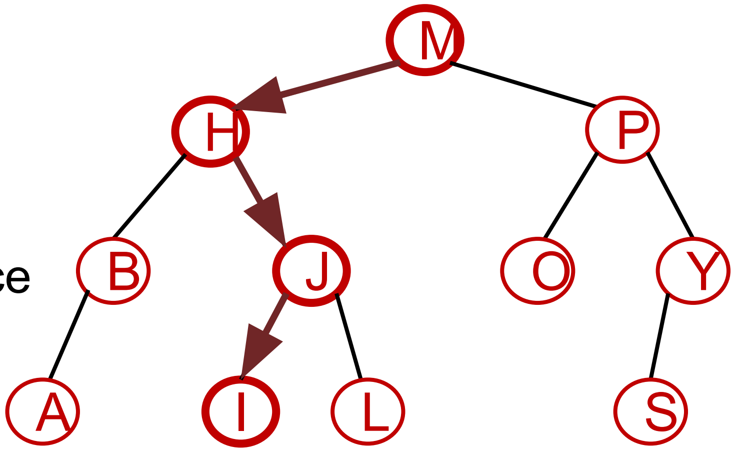
TopHat Question

What's the runtime of (recursive) search in a BST and why?

A. O(n) – because you only iterate once
B. O(2n) – because you go visit both the left and right subtrees
C. O(n/2) – because you incorporate the idea of “bisection” to eliminate half the number of nodes to search at each recursion
D. $O(\log_2 n)$ - because you incorporate the idea of “bisection” to eliminate half the number of nodes to search at each recursion
E. $O(n^2)$ – because recursion makes your runtime quadratic

Searching a BST Recursively Is O($\log_2$N)

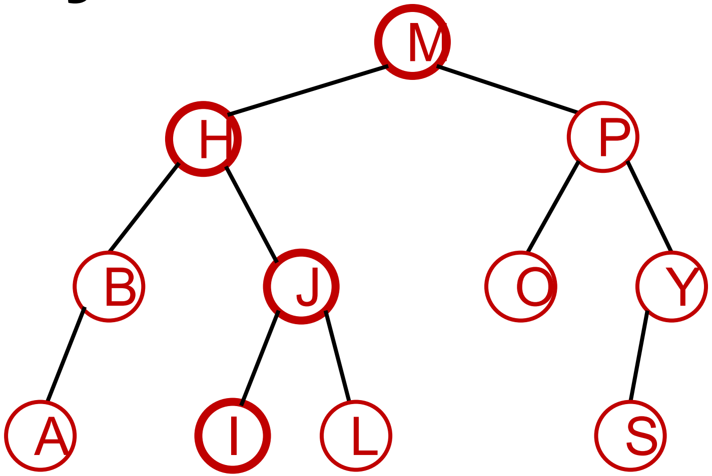
- Search path: start with root **M** and choose path to **I** (for a reasonably balanced tree, **M** will be more or less "in the middle," and left and right subtrees will be roughly the same size)



  - structurally, the height of a reasonably balanced tree with *n* nodes is about $log_2 n$
  - at most, we visit each level of the tree once
  - so, runtime performance of searching is O($\log_2$N) as long as tree is reasonably balanced, which will be true if entry order is reasonably random
  - O($\log_2$N) is much less than N, this is thus much more efficient!

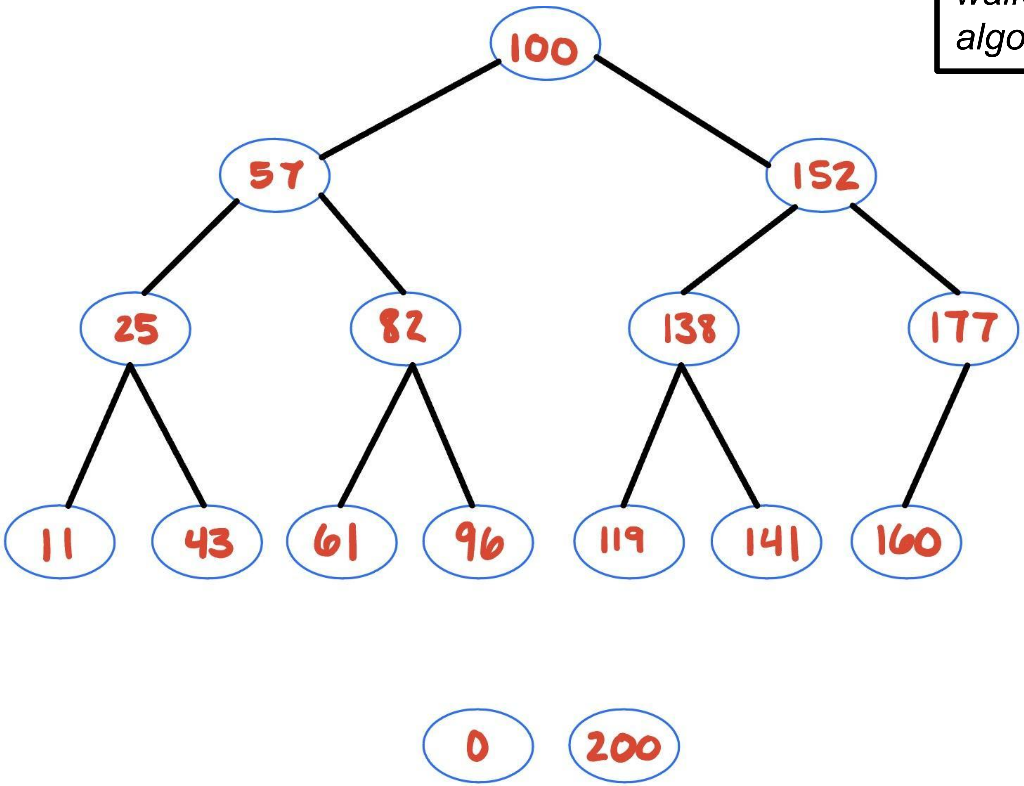
Searching a BST Recursively

```
public Node<Book> search(Book dataToFind) {
    //if data is the thing we're searching for
    if(this.data.compareBooks(dataToFind) == 0) {
        return this.data;
    //if data > dataToFind, can only be in left tree
    } else if(data.compareBooks(dataToFind) > 0) {
        if(this.left != null) {
            return this.left.search(dataToFind);
        }
    //if data < dataToFind, can only be in right tree
    } else if (this.right != null) {
            return this.right.search(dataToFind);
        }
    }
    //Only get here if dataToFind isn't in tree, otherwise would've returned sooner
    return null;
}
```



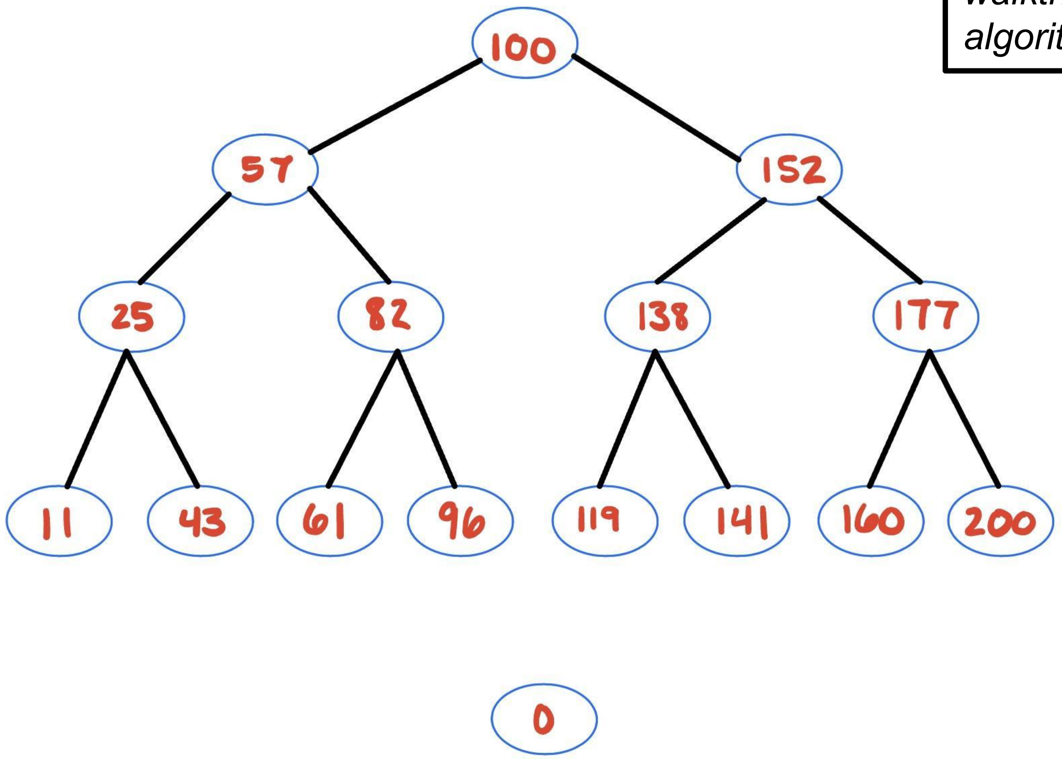
Let's Add to a BST (1/3)

For a step-by-step walkthrough of this algorithm, see slide 112



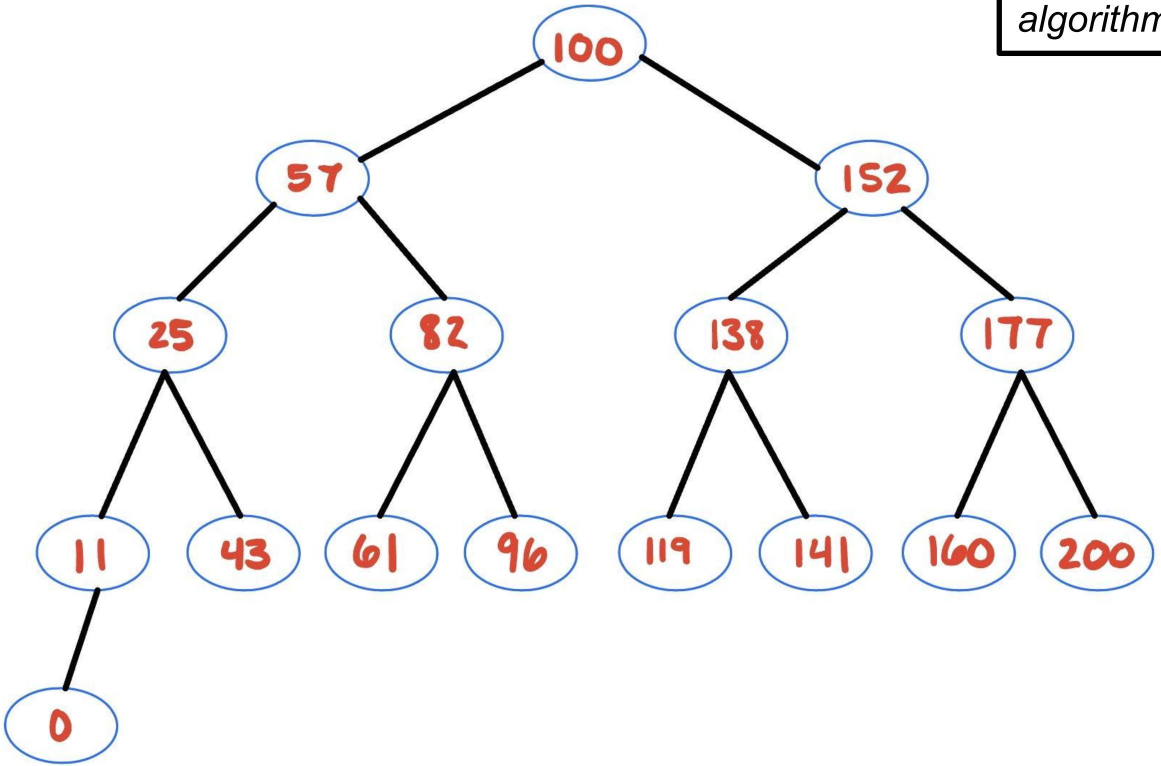
Let’s Add to a BST (2/3)

For a step-by-step walkthrough of this algorithm, see slide 112



Let's Add to a BST (3/3)

For a step-by-step walkthrough of this algorithm, see slide 112



Insertion into a BST

- Search BST starting at root until we find where the data to insert belongs
  - insert data when we reach a `Node` whose appropriate L or R child is `null`
- That `Node` makes a new `Node`, sets the new `Node`'s `data` to the data to insert, and sets child reference to this new `Node`
- Runtime is $O(\log_2 N)$, yay!
  - $O(\log_2 N)$ to search the nearly balanced tree to find the place to insert
  - constant time operations to make new `Node` and link it in

Insertion Code in BST

- Again, we use a “Smart Node” approach and delegate

```
//Tree’s insert delegates to root
public Node<Book> insert(Book newData) {
  //if tree is empty, make first node. No traversal necessary!
  if(this.root == null) {
      this.root = new Node(newData, null); //root’s parent is null
      return this.root;
  } else {
      //delegate to Node’s insert() method
      return this.root.insert(newData);
  }
}
```

Insertion Code in Node

```
public Node<Book> insert(Book newData) {
    if (this.data.compareBooks(newData) > 0) { //newData should be in left subtree
       if(this.left == null) { //left child is null – we’ve found the place to insert!
           this.left = new Node(newData, this);
           return this.left;
       } else { //keep traversing down tree
           return this.left.insert(newData);
       }
    } else { //newData should be in right subtree
       if(this.right == null) { //right child is null-we’ve found the place to insert!
           this.right = new Node(newData, this);
           return this.right;
       } else { //keep traversing down tree
           return this.right.insert(newData);
       }
    }
}
```

Reference to the new Node is passed up the tree so it can be returned by the tree

Notes on Trees (1/2)

- Different insertion order of nodes results in different trees
  - if you insert a node referencing data value of 18 into empty tree, that node will become root
  - if you then insert a node referencing data value of 12, it will become left child of root
  - however, if you insert node referencing 12 into an empty tree, it will become root
  - then, if you insert one referencing 18, that node will become right child of root
  - even with same nodes, different insertion order makes different trees!
  - on average, for reasonably random (unsorted) arrival order, trees will look similar in depth so order doesn't play a major role in runtime

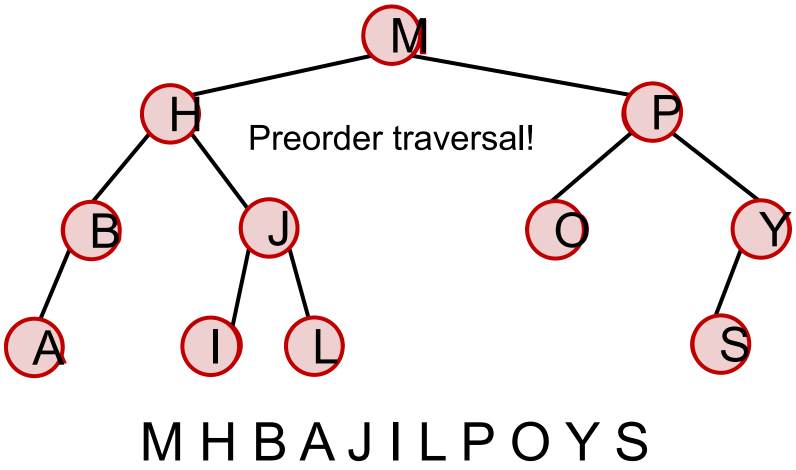
Notes on Trees (2/2)

- When searching for a value, reaching another value that is greater than the one being searched for does not mean that the value being searched for is not present in tree (whereas it does in linked lists!)
  - it may well still be contained in left subtree of node of greater value that has just been encountered
  - thus, where you might have given up in linked lists, **you can't give up here until you reach a leaf** (but depth is roughly $\log_2 N$ for a nearly balanced tree, which is much smaller than N/2!)

Preorder Traversal of BST

- Preorder traversal
  - “pre-order” because self is visited before (“pre-”) visiting children
  - again, use recursion!

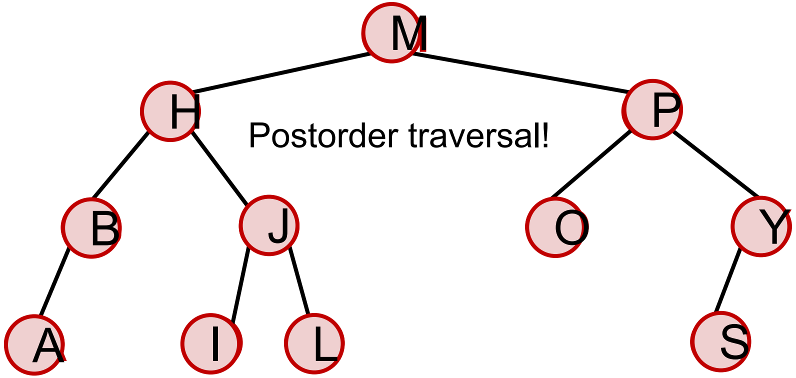
```
public void preOrder() {
    //Check for null children elided
     System.out.println(curr.data);
    this.left.preOrder();
    this.right.preOrder();
}
```



Postorder Traversal of BST

- Postorder traversal
  - “post-order” because self is visited after (“post-”) visiting children
  - again, use recursion!

```
public void postOrder() {
    //Check for null children elided
    this.left.postOrder();
    this.right.postOrder();
    System.out.println(curr.data);
}
```

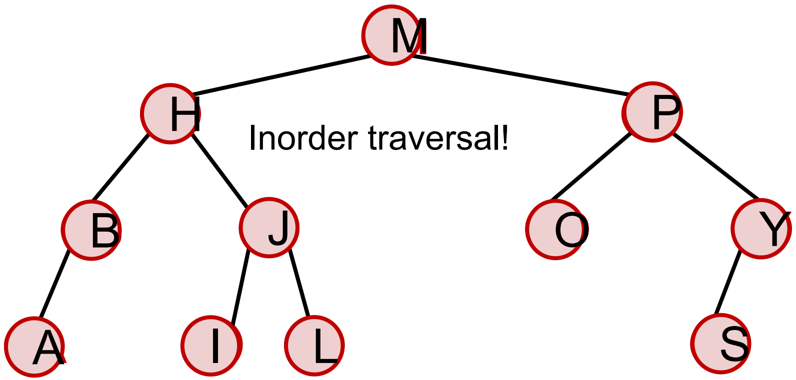


A B I L J H O S Y P M

Inorder Traversal of BST

- Inorder traversal
  - “in-order” because self is visited between (“in-”) visiting children
  - again, use recursion!

```
public void inOrder() {
    //Check for null children elided
    this.left.inOrder();
    System.out.println(curr.data);
    this.right.inOrder();
}
```



A B H I J L M O P S Y

To learn more about the exciting world of trees, take CS200 (CSCI0200): Program Design with Data Structures and Algorithms!

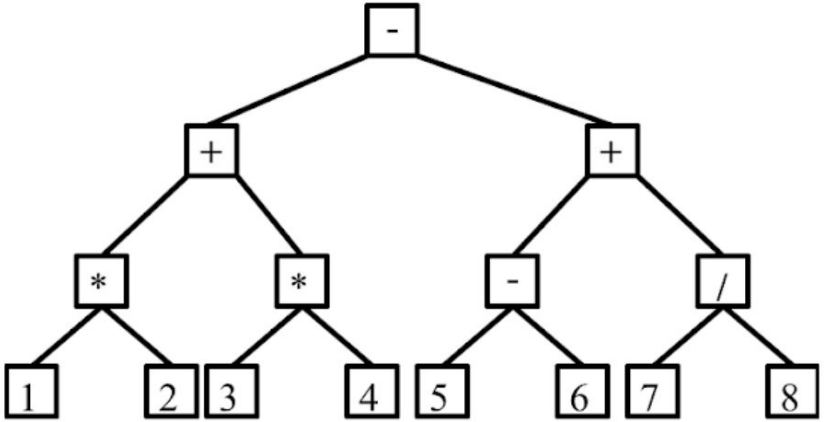
Using Prefix, Infix, Postfix Notation

- When you type an equation into a spreadsheet, you use Infix; when you type an equation into many Hewlett-Packard calculators, you use Postfix, also known as “Reverse Polish Notation,” or “RPN,” after its inventor Polish Logician Jan Lukasiewicz (1924)
- Easier to evaluate Postfix because it has no parentheses and evaluates in a single left-to-right pass
- Use Dijkstra’s 2-stack shunting yard algorithm to convert from user-entered Infix to easy-to-handle Postfix – compile or interpret it on the fly (Covered in optional lecture Dec 6)

Prefix, Infix, Postfix Notation for Arithmetic Expressions

- Infix, Prefix, and Postfix refer to where the operator goes relative to its operands
  - Infix: (fully parenthesized)
    - ((1 * 2) + (3 * 4)) - ((5 - 6) + (7 / 8))
  - Prefix:
    - - + * 1 2 * 3 4 + - 5 6 / 7 8
  - Postfix:
    - 1 2 * 3 4 * + 5 6 - 7 8 / + -

- Graphical representation for equation:



Tree Runtime

- Binary Search Tree has a search of $O(\log_2 n)$ runtime, can we make it faster?
- Could make a ternary tree! (each node has at least 3 children)
  - $O(\log_3 n)$ runtime
- Or a 10-way tree with $O(\log_{10} n)$ runtime
- Let's try the runtime for a search with 1,000,000 nodes
  - $\log_{10} 1{,}000{,}000 = 6$
  - $\log_2 1{,}000{,}00 < 20$, so shallower but broader tree
- Analysis: the logs are not sufficiently different and the comparison (basically an n-way nested if-else-if) is far more time consuming, hence not worth it
- Furthermore, binary tree makes it easy to produce an ordered list

Announcements

- Tetris deadlines
  - early handin: Saturday 11/12
  - on-time handin: Monday 11/14
  - late handin: Wednesday 11/16

- HTA Hours Friday 3-4pm (as always!) in Friedman 101
  - come talk to us about which FP to do!

Wondering how to make a *generic* BST that can store more than just books?

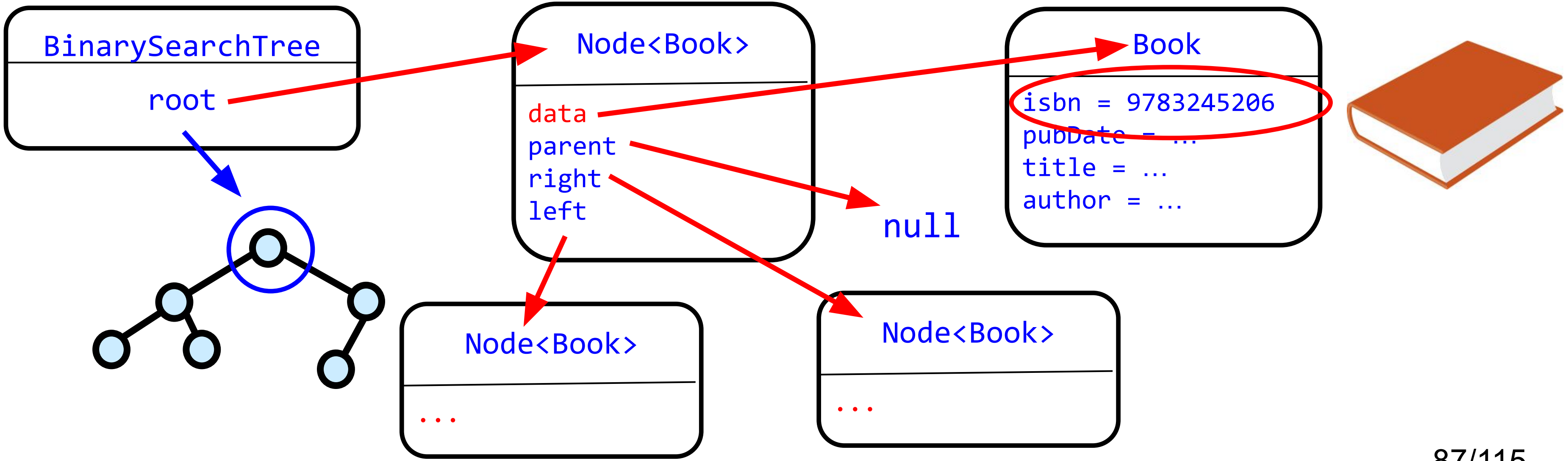
(yes! there's a way!)

Appendix

- Generic BST
- Searching Simulation
- Insertion Demonstration

Nodes, data, and keys

- data is a composite that can contain many properties,
- one of which is a key that Nodes are sorted by (here, ISBN #) – but how do we compare Nodes to sort them?



Java's `Comparable<Type>` interface (1/3)

- Previously we used `==` to check if two things are equal
  - this only works correctly for primitive data types (e.g., `int`), or when we are comparing two variables referencing the exact same object
  - to compare `String`s, need a different way to compare things

- We can implement the `Comparable<Type>` generic interface provided by Java

- It specifies the `compareTo` method, which returns an `int`

- Why don't we just use ==, even when using something like ISBN, which is an `int`?
  - can treat ISBNs as `int`s and compare them directly, but more generally we implement the `Comparable<Type>` interface, which could easily accommodate comparing `String`s, such as author or title, or any other property

Java's `Comparable<Type>` interface (2/3)

- The `Comparable<Type>` interface is specialized (think of it as parameterized) using generics

```
public interface Comparable<Type> {
    int compareTo(Type toCompare);
}
```

- Call `compareTo` on a variable of same type as specified in implementator of interface (`Book`, in our case)
    - `currentBook.compareTo(bookToFind);`

Java's `Comparable<Type>` interface (3/3)

- `compareTo` method must return an `int`
  - **negative** if element on which `compareTo` is called is *less* than element passed in as the parameter of the search
  - **0** if element is *equal* to element passed in
  - **positive** if element is *greater* than element passed in
  - sign of `int` returned is all-important, magnitude is not and is implementation dependent
- `compareTo` not only used for numerical comparisons–it could be used for alphabetical or geometric comparisons as well–depends on how you implement `compareTo`

“Comparable” Book Class

- Recall format for `compareTo`:
  - `elementA.compareTo(elementB)`
- Book class now implements `Comparable<Book>`
  - this means we can compare books, using `bookA.compareTo(bookB)`
- `compareTo` is defined according to these specifications
  - returns number that is `<0`, `0` or `>0`, depending on the ISBN numbers
  - <0 if stored `this.isbn` < `toCompare`

```
public class Book implements Comparable<Book> {
    // variable declarations, e.g., isbn, elided
    public Book(String author, String title,
                      int isbn){
        //variable initializations elided
    }

    public int getISBN(){
        return this.isbn;
    }

    //other methods elided

    //compare isbn of book passed in to stored one
    @Override
    public int compareTo(Book toCompare){
        return (this.isbn - toCompare.getISBN());
    }
}
```

BST Class (2/4)

In our example use Book as Type

- Using keyword extends in this way ensures that Type implements Comparable<Type>
  - note nested <>
  - nested <> to show it modifies Type and not the class
- All elements stored in MyLinkedList must now have compareTo method for Type; thus restricts generic

```java
public class BinarySearchTree<Type extends
                             Comparable<Type>> {

    private Node<Type> root;

    public BinarySearchTree(Type data) {
        //Root of the tree
        this.root = new Node(data, null);
    }

    // other methods shown next slide

}
```

BST Class (3/4)

```java
public class BinarySearchTree<Type extends
  Comparable<Type>> {

    private Node<Type> root;

    public BinarySearchTree(Type data) {
        //Root of the tree
        this.root = new Node(data, null);
    }

    public void insert(Type newData) {
      // . . .
    }
```

```java
    //class continued
    public void remove(Type dataToRemove) {
        // . . .
    }

    public Node<Type> search(Type dataToFind) {
        // . . .
    }

    public int size() {
        // . . .
    }

} // end of class
```

BST Class (4/4)

- Our implementations of `LinkedList`s, `Stack`s, and `Queue`s are “smart” data structures that chain “dumb” nodes together
  - the lists did all the work by maintaining previous and current pointers and did the operations to search for, insert, and remove information – thus, nodes were essentially data containers
- Now we will use a “dumb” tree with “smart” nodes that will delegate using recursion
  - tree will delegate action (such as searching, inserting, etc.) to its root, which will then delegate to its appropriate child, and so on
  - creates specialized `Node` class that stores its data, parent, and children, and can perform operations such as `insert` and `remove`

BST: Node Class (1/3)

- “Smart” Node includes the following methods:

```
// pass in entire data item, containing key, so compareTo() will work
public Node<Type> search(Type dataToFind);
public Node<Type> insert(Type newData);

/* remove deletes Node pointing to dataToRemove, which contains key;
removing Node also will remove the matched data element instance unless
there’s another reference to it */
public Node<Type> remove(Type dataToRemove);
```

- Plus setters and getters of instance variables, defined in the next slides …

BST: Node Class (2/3)

- Nodes have a maximum of two non-null children that hold data implementing Comparable<Type>
  - four instance variables: data, parent, left, and right, with each having a get and set method.
  - data represents the data that Node stores. It also contains the key attribute that Nodes are sorted by – we'll make a Tree that stores Books
  - parent represents the direct parent (another Node) of Node–only used in remove method
  - left represents Node's left child and contains a subtree, all of whose data is less than Node's data
  - right represents Node's right child and contains a subtree, all of whose data is greater than Node's data
  - arbitrarily select which child should contain data equal to Node's data

BST: Node Class (3/3)

```
public class Node<Type implements Comparable<Type>> {
    private Type data;
    private Type parent;
    private Node<Type> left;
    private Node<Type> right;
    public Node(Type data, Node<Type> parent){ //construct a leaf node as default
        this.data = data;
        this.parent = parent;
        //child ptrs null for leaf nodes; set for internal nodes when child is created
        this.left = null;
        this.right = null;
    }
   // will define other methods in next slides…
}
```

Smart Node Approach

- `BinarySearchTree` is “dumb,” so it delegates to root, which in turn will delegate recursively to its left or right child, as appropriate

```
// search method for entire BinarySearchTree:
public Node<Type> search(dataToFind) {
            return this.root.search(dataToFind);
}
```

- Smart node approach makes our code clean, simple and elegant
  - non-recursive method is much messier, involving explicit bookkeeping of which node in the tree we are currently processing
    - we used the non-recursive method for sorted linked lists, but trees are more complicated, and recursion is easier – a tree is composed of subtrees!

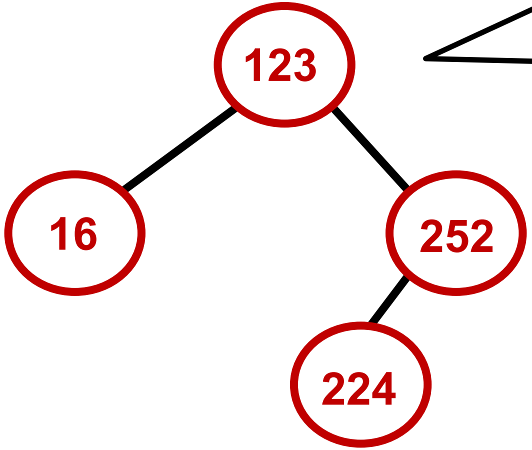
Appendix

- Generic BST
- Searching Simulation
- Insertion Demonstration

Searching Simulation (animated)

- What if we want to know if 224 is in Tree?
- Tree says:

"Hey Root! Ya got 224?"

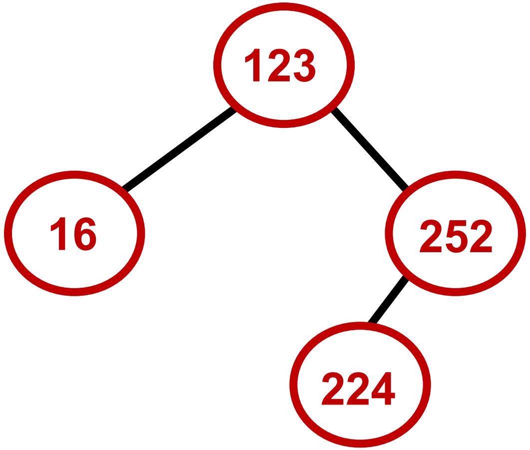


123 says:

"Let's see. I'm not 224. But if 224 is in tree, since it's larger, it would be to my right. I'll ask my right child and return its answer."

Searching Simulation (animated)

- What if we want to know if **224** is in Tree?

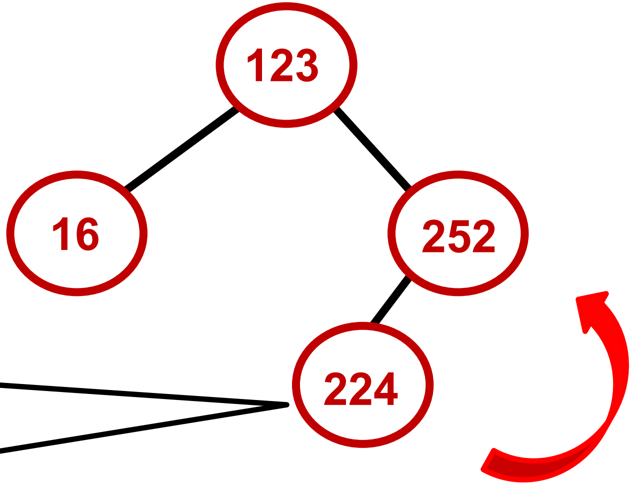


252 says:

"I'm not **224**. I better ask my left child and return its answer."

Searching Simulation (animated)

- What if we want to know if **224** is in Tree?



224 says:

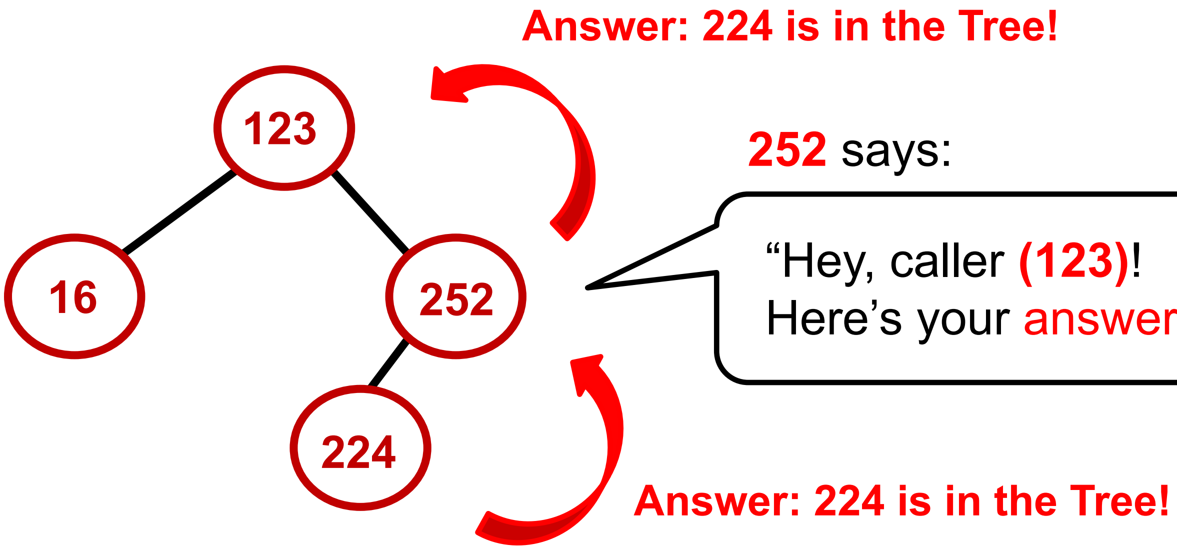
"**224**? That's me!
Hey, caller **(252)**
here's your
answer."

(returning node indicates that query is in tree)

Answer: 224 is in the Tree!

Searching Simulation (animated)

- What if we want to know if **224** is in Tree?

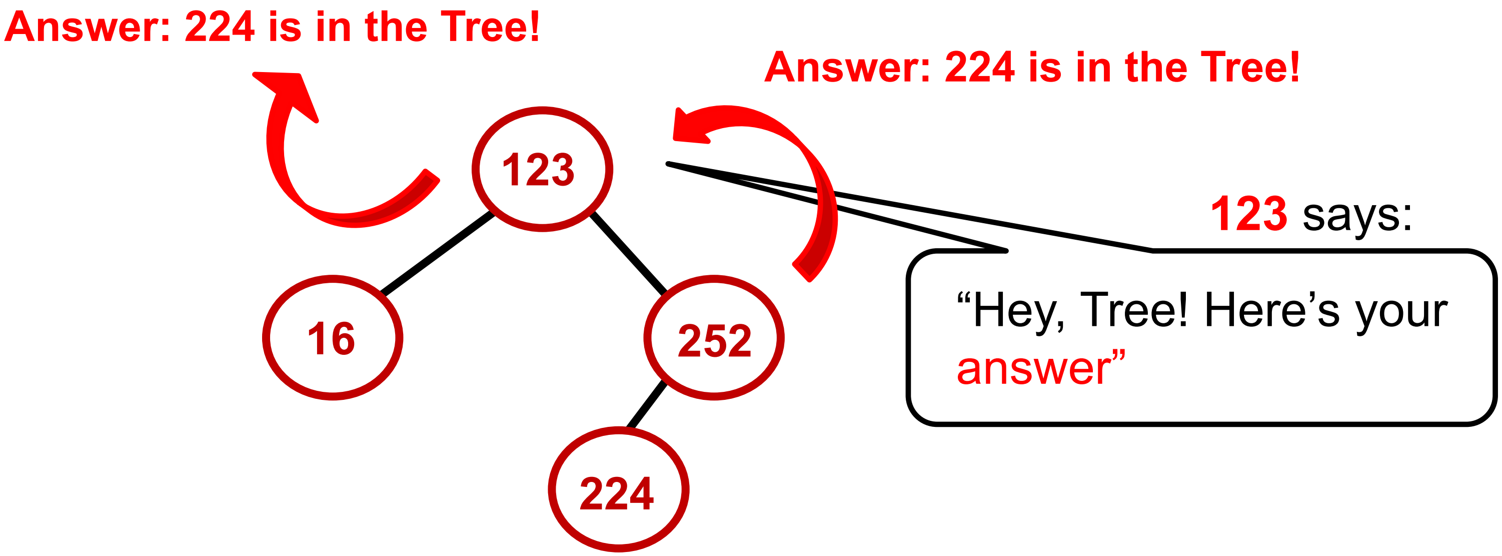


252 says:

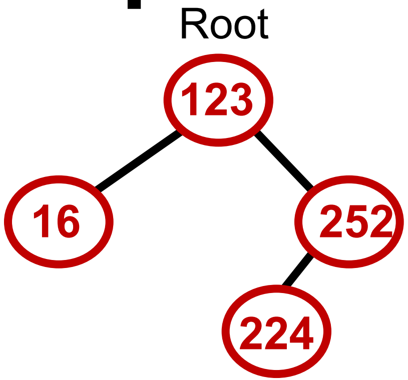
"Hey, caller **(123)**!
Here's your answer."

Searching Simulation (animated)

- What if we want to know if **224** is in Tree?



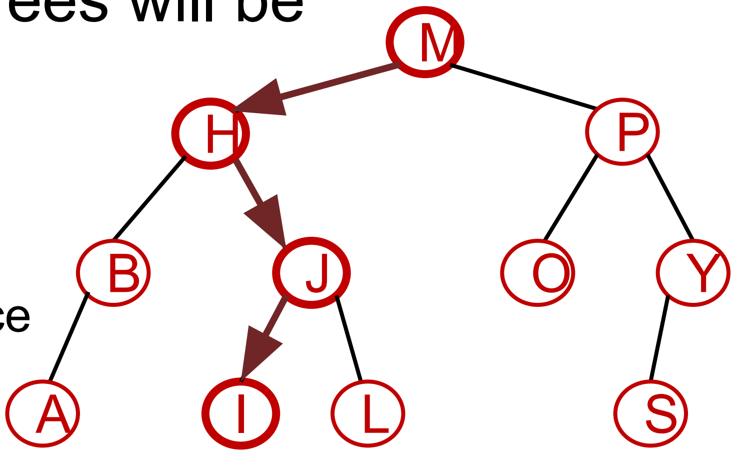
Searching Simulation - Recap



- What if we want to know if **224** is in Tree?
- Tree says “Hey Root! Ya got **224**?”
- **123** says: “Let’s see. I’m not **224**. But if **224** is in tree, it would be to my right. I’ll ask my right child and return its answer.”
- **252** says: “I’m not **224**, it’s smaller than me. I better ask my left child and return its answer.”
- **224** says: “**224**? That’s me! Hey, caller **(252)** here’s your answer.” (returning node indicates that query is in tree)
- **252** says: “Hey, caller **(123)**! Here’s your answer.”
- **123** says: “Hey, Tree! Here’s your answer.”

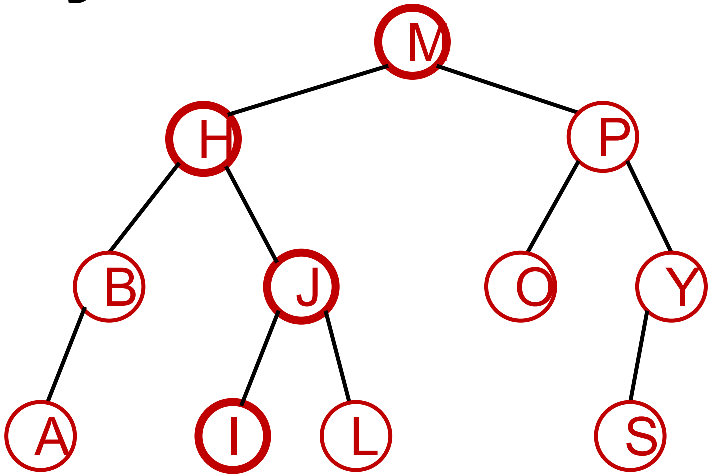
Searching a BST Recursively Is $O(\log_2 N)$

- Search path: start with root **M** and choose path to **I** (for a reasonably balanced tree, **M** will be more or less "in the middle," and left and right subtrees will be roughly the same size)
  - structurally, the height of a reasonably balanced tree with n nodes is about $\log_2 n$
  - at most, we visit each level of the tree once
  - so, runtime performance of searching is $O(\log_2 N)$ as long as tree is reasonably balanced, which will be true if entry order is reasonably random (slide 87)



Searching a BST Recursively

```
public Node<Type> search(Type dataToFind) {
    //if data is the thing we're searching for
    if(this.data.compareTo(dataToFind) == 0) {
        return this.data;
    //if data > dataToFind, can only be in left tree
    } else if(data.compareTo(dataToFind) > 0) {
        if(this.left != null) {
            return this.left.search(dataToFind);
        }
    //if data < dataToFind, can only be in right tree
    } else if (this.right != null) {
            return this.right.search(dataToFind);
        }
    }
    //Only get here if dataToFind isn't in tree, otherwise would've returned sooner
    return null;
}
```



Appendix

- Generic BST
- Searching Simulation
- Insertion Demonstration

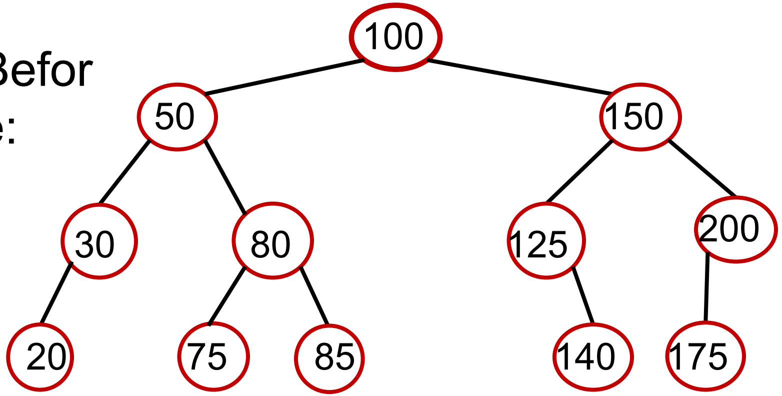
Insertion into a BST(1/2)

- Search BST starting at root until we find where the data to insert belongs
  - insert data when we reach a `Node` whose appropriate L or R child is `null`
- That `Node` makes a new `Node`, sets the new `Node`'s `data` to the data to insert, and sets child reference to this new `Node`
- Runtime is $O(\log_2 N)$, yay!
  - $O(\log_2 N)$ to search the nearly balanced tree to find the place to insert
  - constant time operations to make new `Node` and link it in

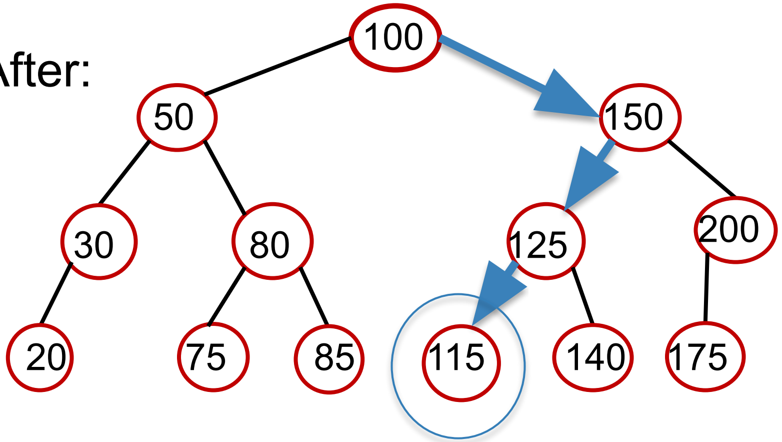
Insertion into a BST(2/2)

- Example: Insert **115**

Befor
e:



After:



Insertion Code in BST

- Again, we use a “Smart Node” approach and delegate

```
//Tree’s insert delegates to root
public Node<Type> insert(Type newData) {
  //if tree is empty, make first node. No traversal necessary!
  if(this.root == null) {
      this.root = new Node(newData, null); //root’s parent is null
      return this.root;
  } else {
      //delegate to Node’s insert() method
      return this.root.insert(newData);
  }
}
```

Insertion Code in Node

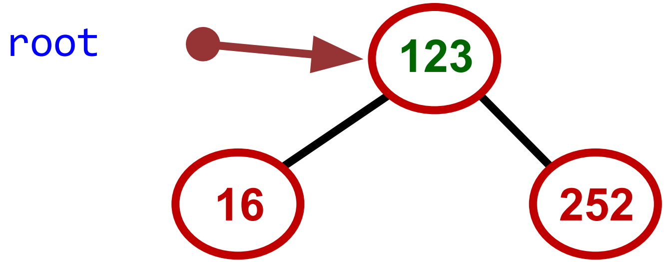
```
public Node<Type> insert(Type newData) { //insert method continued!
    if (this.data.compareTo(newData) > 0) { //newData should be in left subtree
       if(this.left == null) { //left child is null – we’ve found the place to insert!
           this.left = new Node(newData, this);
           return this.left;
       } else { //keep traversing down tree
           return this.left.insert(newData);
       }
    } else { //newData should be in right subtree
       if(this.right == null) { //right child is null-we’ve found the place to insert!
           this.right = new Node(newData, this);
           return this.right;
       } else { //keep traversing down tree
           return this.right.insert(newData);
       }
    }
}
```

Reference to the new Node is passed up the tree so it can be returned by the tree

Insertion Simulation (1/4)

- Insert: **224**
- First call `insert` in BST:

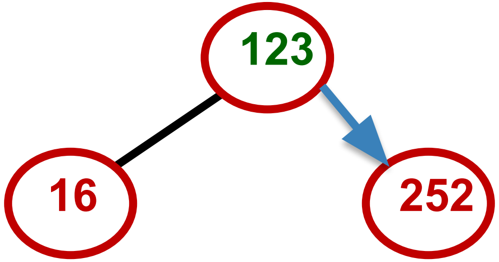
```
this.root = this.root.insert(newData);
```



Insertion Simulation (2/4)

- 123 says: “I am less than 224. I’ll let my right child deal with it.”

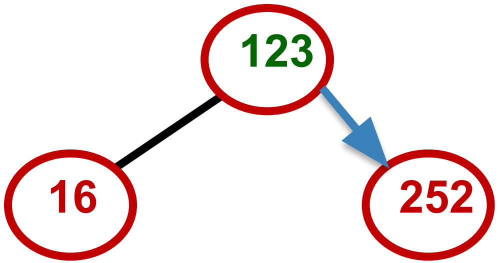
```
if (this.data.compareTo(newData) > 0) {
       //code for inserting left elided
} else {
       if(this.right == null) {
           //code for inserting with null right child elided
       } else {
       return this.right.insert(newData);
       }
}
```



Insertion Simulation (3/4)

- 252 says: "I am greater than 224. I'll pass it on to my left child – but my left child is `null`!"

```
if (this.data.compareTo(newData) > 0) {
    if(this.left == null) {
        this.left = new Node(newData, this);
        return this.left;
    } else {
        //code for continuing traversal elided
    }
}
```

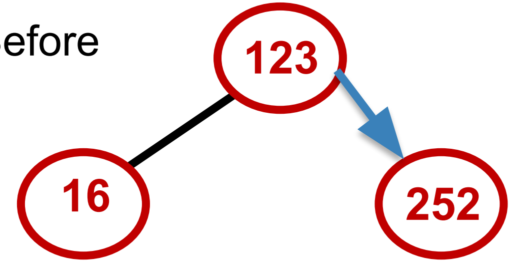


Insertion Simulation (4/4)

- 252 says: "You belong as my left child, 224. Let me make a node for you, make this new node your home, and set that node as my left child. Lastly, I will return a pointer to the new left node". (And each node, as its recursive invocation ends, passes the pointer to the new 224 node up to its parent, eventually up to whatever method called on the tree's search)

```
this.left = new Node(newData, this);
return this.left;
```

Before



After