Structure of an induction proof

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To prove: [write out statement here]	To prove: the sum of the numbers $1 + 2 + \ldots + n$ is $n(n+1)/2$ for every positive integer n.
We'll prove this by induction. Let $P(n)$ be the statement that *fill in predicate here*.	We'll prove this by induction. Let $P(n)$ be the statement that $$1 + 2 + \ldots + n = n(n + 1)/2.$$
Let S be the set of positive integers k such that $P(k)$ is true.	Let S be the set of positive integers k such that $P(k)$ is true.
We'll show first that $1 \in S$, and second, that if $k \in S$, then $k + 1 \in S$. Together, these imply that all positive integers are in S, and we will be done.	We'll show first that $1 \in S$, and second, that if $k \in S$, then $k + 1 \in S$. Together, these imply that all positive integers are in S, and we will be done.
Base case: To show that $1 \in S$, we'll write out $P(1)$: *write out P(1) here, and give an explanation of why it's true.*	Base case: To show that $1 \in S$, we'll write out $P(1)$: $1 + \ldots + 1 = 1(1 + 1)/2$. The left hand side is just the single number 1; the right hand side simplifies to $1 \cdot 2/2 = 1$, and we see they're equal.
So $P(1)$ is true, and $1 \in S$.	So $P(1)$ is true, and $1 \in S$.
Inductive step: For some fixed but unknown positive integer k, we'll assume $P(k)$ is true, i.e., we'll assume that *write out* $P(k)$ *here*.	Inductive step: For some fixed but unknown positive integer k, we'll assume $P(k)$ is true, i.e., we'll assume that $$1 + 2 + \ldots + k = k(k + 1)/2.$$

We'll then use this to prove $P(k+1)$, i.e., to prove *write out* $P(k+1)$ *here.*

We'll then use this to prove $P(k+1)$, i.e., to prove that

$$\begin{aligned} 1+2+\ldots+k+1 = & \quad (k+1)((k+1)+1)/2 \\ = & \quad (k+1)(k+2)/2. \end{aligned}$$

Start from $P(k)$ *and argue the truth of* $P(k+1)$.

We've assumed that

$$1+2+\ldots+k = k(k+1)/2.$$

Adding $k+1$ to each side yields

$$1+2+\ldots+k+(k+1) = k(k+1)/2+(k+1).$$

Doing a little algebra on the right hand side, we get

$$\begin{aligned} 1+2+\ldots+(k+1) &= k(k+1)/2+2(k+1)/2 \\ &= [k(k+1)+2(k+1)]/2 \\ &= [(k+2)(k+1)]/2 \\ &= (k+1)(k+2)/2, \end{aligned}$$

which is exactly the statement $P(k+1)$, which we promised to prove.

We conclude, by induction, that S contains all positive integers, so $P(n)$ is true for every positive integer n.

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