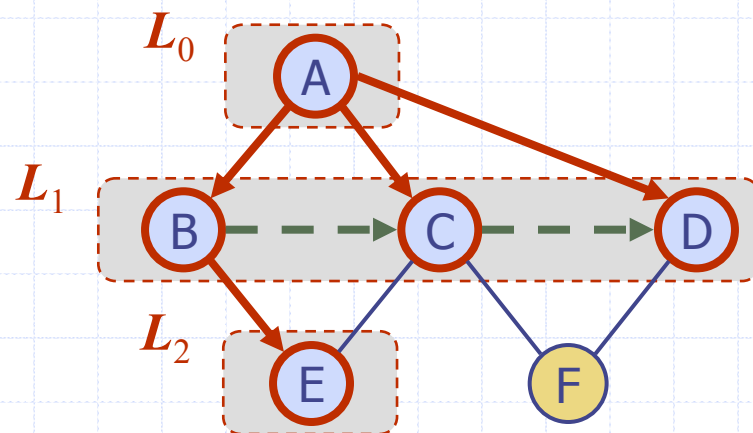


# Breadth-First Search



# Outline and Reading

## ◆ Breadth-first search (§13.3.3)

- Algorithm
- Example
- Properties
- Analysis
- Applications

## ◆ DFS vs. BFS

- Comparison of applications
- Comparison of edge labels

# Breadth-First Search

- ◆ Breadth-first search (BFS) is a general technique for traversing a graph
- ◆ A BFS traversal of a graph  $G$ 
  - Visits all the vertices and edges of  $G$
  - Determines whether  $G$  is connected
  - Computes the connected components of  $G$
  - Computes a spanning forest of  $G$
- ◆ BFS on a graph with  $n$  vertices and  $m$  edges takes  $O(n + m)$  time
- ◆ BFS can be further extended to solve other graph problems
  - Find and report a path with the minimum number of edges between two given vertices
  - Find a simple cycle, if there is one

# BFS Algorithm

- ◆ The algorithm uses a mechanism for setting and getting “labels” of vertices and edges

## Algorithm *BFS(G)*

**Input** graph  $G$

**Output** labeling of the edges and partition of the vertices of  $G$

```
for all  $u \in G.vertices()$ 
   $setLabel(u, UNEXPLORED)$ 
for all  $e \in G.edges()$ 
   $setLabel(e, UNEXPLORED)$ 
for all  $v \in G.vertices()$ 
  if  $getLabel(v) = UNEXPLORED$ 
     $BFS(G, v)$ 
```

## Algorithm *BFS(G, s)*

$L_0 \leftarrow$  new empty sequence

$L_0.insertLast(s)$

$setLabel(s, VISITED)$

$i \leftarrow 0$

**while**  $\neg L_i.isEmpty()$

$L_{i+1} \leftarrow$  new empty sequence

**for all**  $v \in L_i.elements()$

**for all**  $e \in G.incidentEdges(v)$

**if**  $getLabel(e) = UNEXPLORED$

$w \leftarrow opposite(v, e)$

**if**  $getLabel(w) = UNEXPLORED$

$setLabel(e, DISCOVERY)$

$setLabel(w, VISITED)$

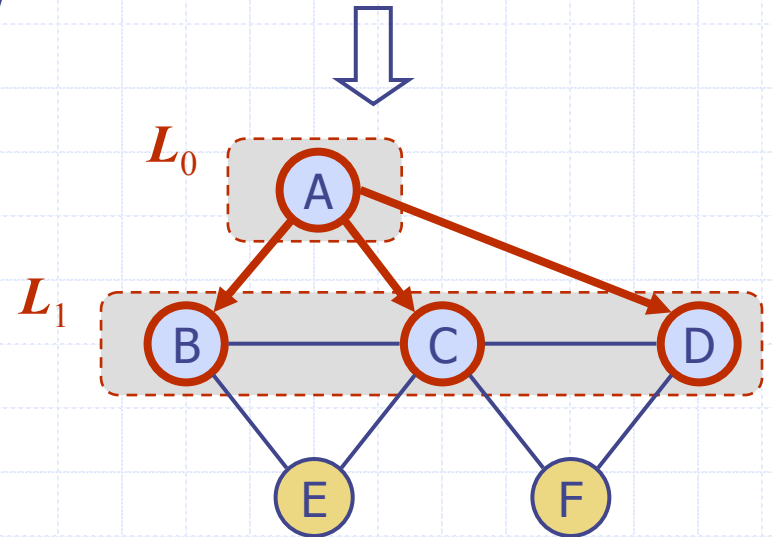
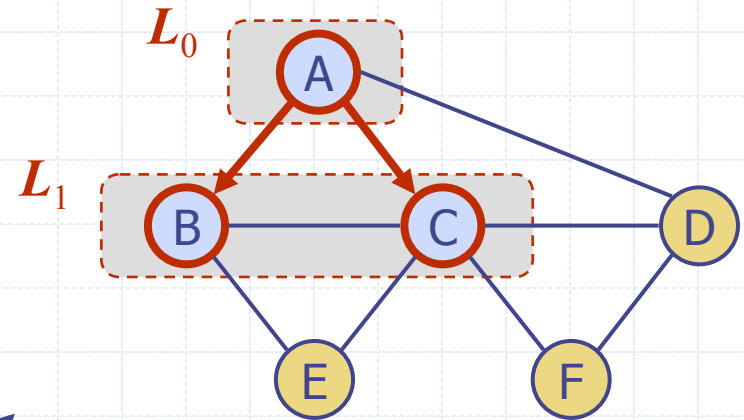
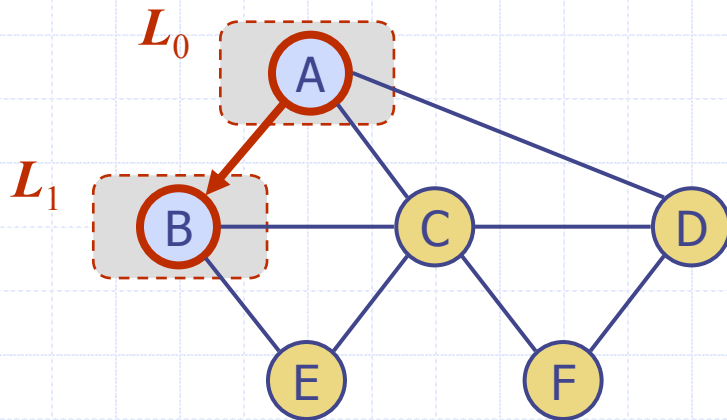
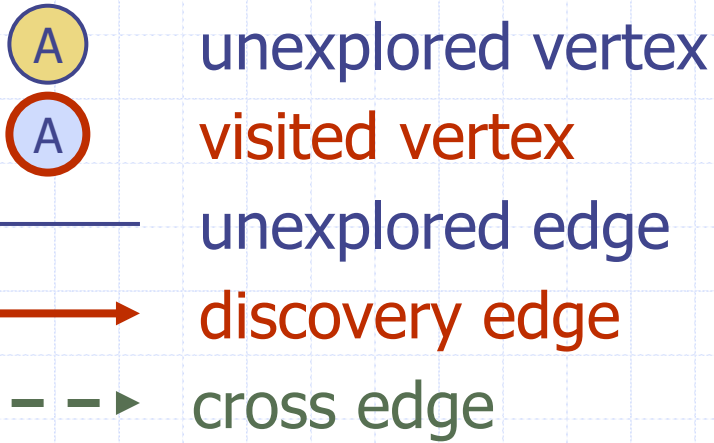
$L_{i+1}.insertLast(w)$

**else**

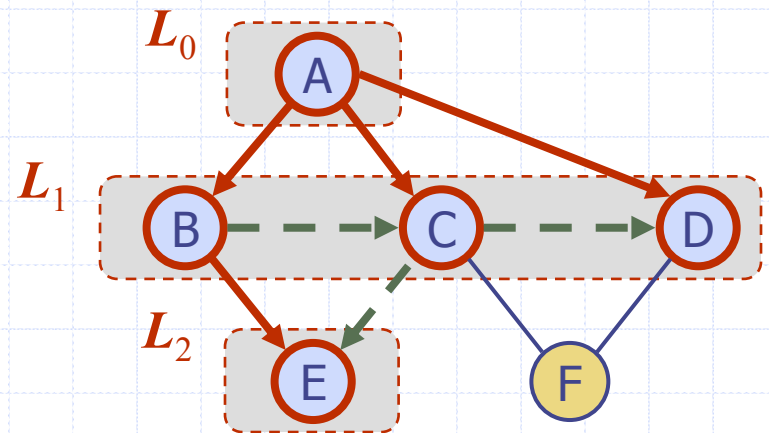
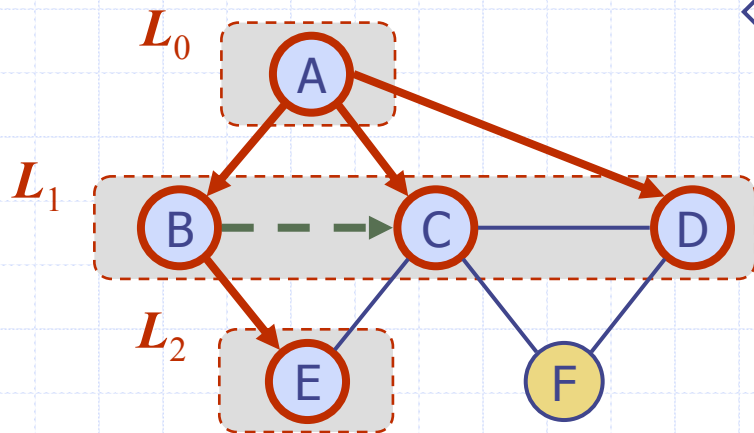
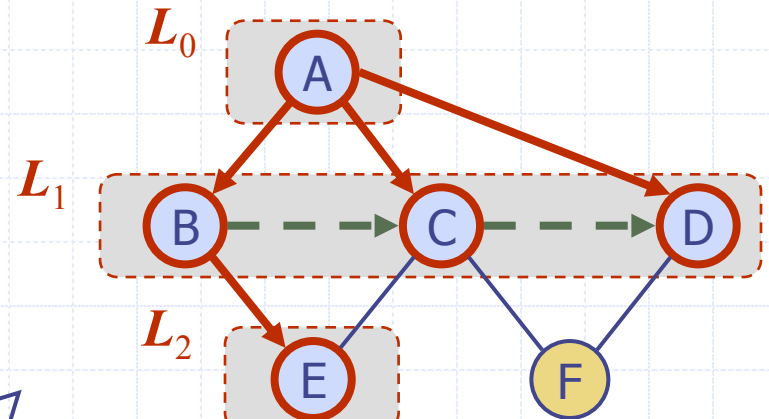
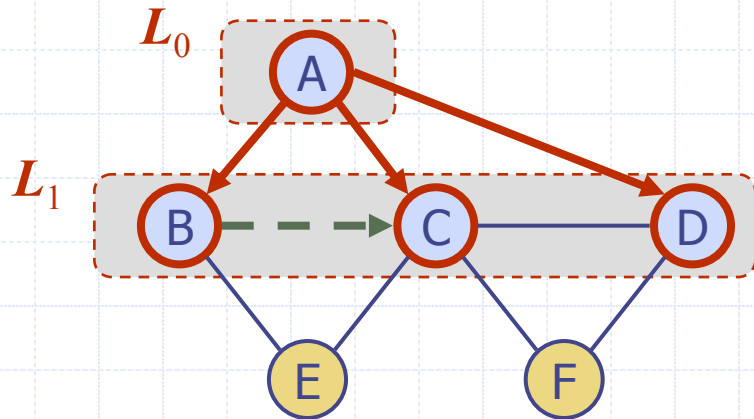
$setLabel(e, CROSS)$

$i \leftarrow i + 1$

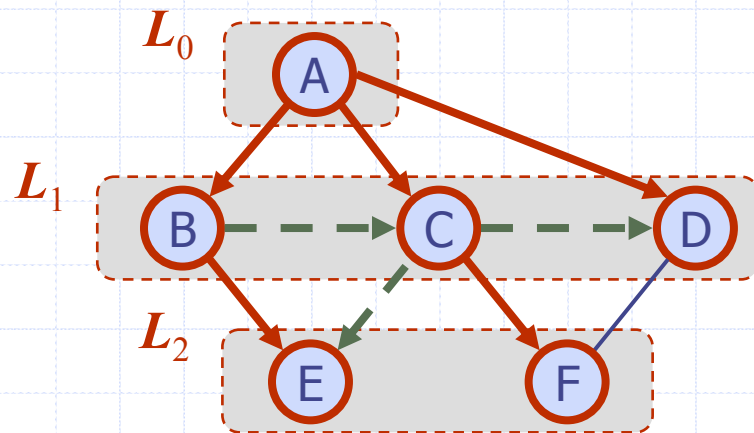
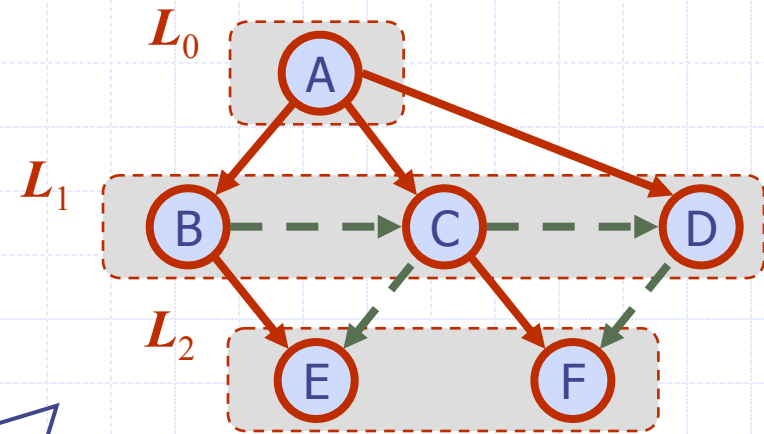
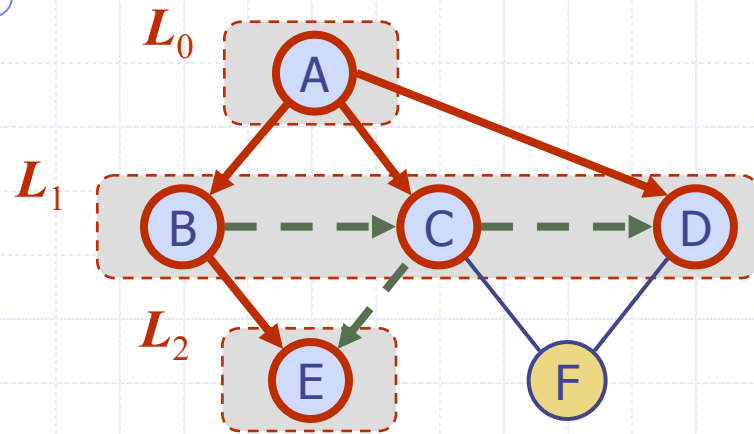
# Example



# Example (cont.)



# Example (cont.)



# Properties

## Notation

$G_s$ : connected component of  $s$

## Property 1

$BFS(G, s)$  visits all the vertices and edges of  $G_s$

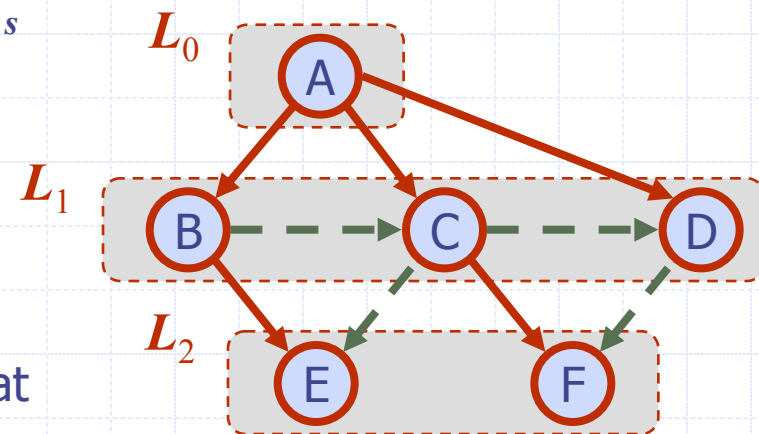
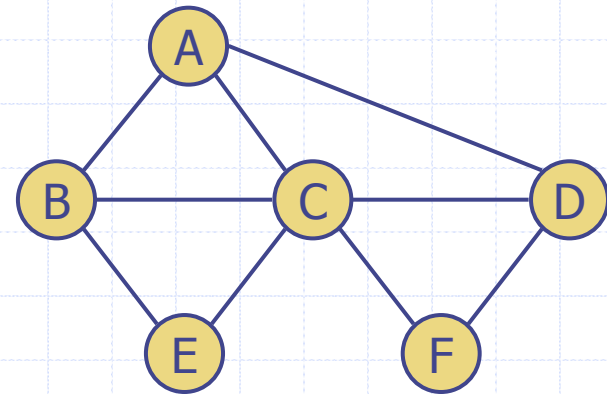
## Property 2

The discovery edges labeled by  $BFS(G, s)$  form a spanning tree  $T_s$  of  $G_s$

## Property 3

For each vertex  $v$  in  $L_i$

- The path of  $T_s$  from  $s$  to  $v$  has  $i$  edges
- Every path from  $s$  to  $v$  in  $G_s$  has at least  $i$  edges



# Analysis

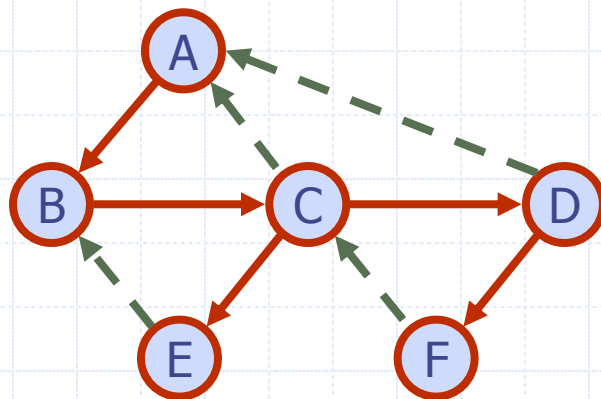
- ◆ Setting/getting a vertex/edge label takes  $O(1)$  time
- ◆ Each vertex is labeled twice
  - once as UNEXPLORED
  - once as VISITED
- ◆ Each edge is labeled twice
  - once as UNEXPLORED
  - once as DISCOVERY or CROSS
- ◆ Each vertex is inserted once into a sequence  $L_i$
- ◆ Method incidentEdges is called once for each vertex
- ◆ BFS runs in  $O(n + m)$  time provided the graph is represented by the adjacency list structure
  - Recall that  $\sum_v \deg(v) = 2m$

# Applications

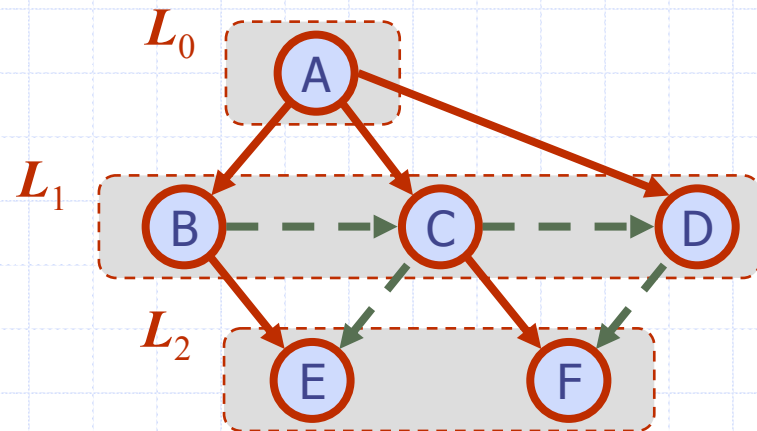
- ◆ Using the template method pattern, we can specialize the BFS traversal of a graph  $G$  to solve the following problems in  $O(n + m)$  time
  - Compute the connected components of  $G$
  - Compute a spanning forest of  $G$
  - Find a simple cycle in  $G$ , or report that  $G$  is a forest
  - Given two vertices of  $G$ , find a path in  $G$  between them with the minimum number of edges, or report that no such path exists

# DFS vs. BFS

Applications	DFS	BFS
Spanning forest, connected components, paths, cycles	✓	✓
Shortest paths		✓
Biconnected components	✓	



DFS

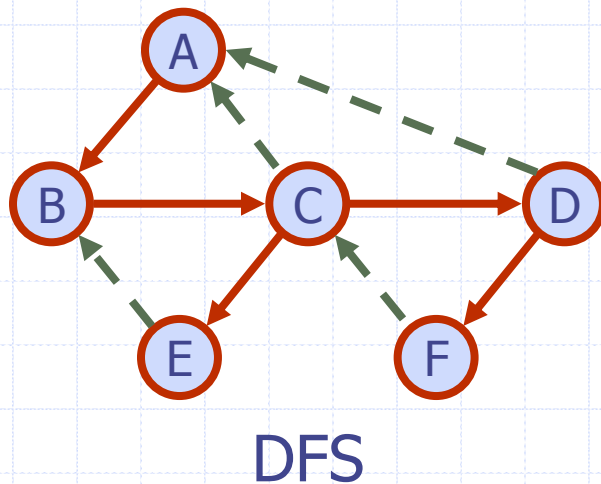


BFS

# DFS vs. BFS (cont.)

## Back edge ( $v, w$ )

- $w$  is an ancestor of  $v$  in the tree of discovery edges



## Cross edge ( $v, w$ )

- $w$  is in the same level as  $v$  or in the next level in the tree of discovery edges

