CSCI 0220 **Discrete Structures and Probability** B. Raphael

Homework 1

Solution Key

Problem 1.1

*Using a **truth table** decide if each of the following expressions is a contradiction, a tautology, or neither.*

a. $[(r \wedge \sim q) \vee (\sim p \wedge \sim r)] \wedge (q \wedge r)$

b. $[(\sim p) \vee (p \wedge \sim q)] \vee [(r \wedge p) \vee (\sim r)]$

c. $[(p \vee q) \wedge (q \wedge \sim r)] \wedge [(\sim r \vee p) \wedge (\sim q)]$

a. contradiction

p	q	r	$r \wedge \sim q$	$\sim p \wedge \sim r$	$q \wedge r$	$(r \wedge \sim q \vee$ $(\sim p \wedge \sim r)$	$[(r \wedge \sim q) \vee (\sim p \wedge \sim r)]$ $\wedge (q \wedge r)$
T	T	T	F	F	T	F	F
T	T	F	F	F	F	F	F
T	F	T	T	F	F	T	F
T	F	F	F	F	F	F	F
F	T	T	F	F	T	F	F
F	T	F	F	T	F	T	F
F	F	T	T	F	F	T	F
F	F	F	F	T	F	T	F

b. tautology

p	q	r	$\sim p$	$p \wedge \sim q$	$r \wedge p$	$\sim r$	$(\sim p) \vee$ $(p \wedge \sim q)$	$(r \wedge p)$ $\vee(\sim r)$	$[(\sim p) \vee (p \wedge \sim q)] \vee$ $[(r \wedge p) \vee (\sim r)]$
T	T	T	F	F	T	F	F	T	T
T	T	F	F	F	F	T	F	T	T
T	F	T	F	T	T	F	T	T	T
T	F	F	F	T	F	T	T	T	T
F	T	T	T	F	F	F	T	F	T
F	T	F	T	F	F	T	T	T	T
F	F	T	T	F	F	F	T	F	T
F	F	F	T	F	F	T	T	T	T

c. contradiction

p	q	r	$p \vee q$	$q \wedge \sim r$	$\sim r \vee p$	$\sim q$	$(p \vee q) \wedge$ $(q \wedge \sim r)$	$(\sim r \vee p)$ $\wedge (\sim q)$	$[(p \ \vee q) \wedge (q \wedge \sim r)]$ $\wedge [(\sim r \vee p) \wedge (\sim q)]$
T	T	T	T	F	T	F	F	F	F
T	T	F	T	T	T	F	T	F	F
T	F	T	T	F	T	T	F	T	F
T	F	F	T	F	T	T	F	T	F
F	T	T	T	F	F	F	F	F	F
F	T	F	T	T	T	F	T	F	F
F	F	T	F	F	F	T	F	F	F
F	F	F	F	F	T	T	F	T	F

Problem 1.2

*Simplify each expression and cite the **specific rule** used.*

a. $(p \ \rightarrow \sim q) \wedge (\sim q \vee \sim p)$

b. $\sim (p \wedge r) \vee (\sim q \wedge r)$

c. $\sim q \wedge (\sim p \wedge [\sim (p \wedge \sim r) \vee (\sim p \wedge q)])$

For Problem A:

1. $(p \ \rightarrow \sim q) \wedge (\sim q \vee \sim p)$ (given)
2. $(\sim p \vee \sim q) \wedge (\sim q \vee \sim p)$ (definition of $\rightarrow$)
3. $(\sim p \vee \sim q) \wedge (\sim p \vee \sim q)$ (commutative)
4. $\sim p \vee \sim q$ (idempotent)

For Problem B:

1. $\sim (p \wedge r) \vee (\sim q \wedge r)$ (given)
2. $\sim p \vee \sim r \vee (\sim q \wedge r)$ (de Morgan's)
3. $\sim p \vee [(\sim r \vee \sim q) \wedge (\sim r \vee r)]$ (distributive)
4. $\sim p \vee [(\sim r \vee \sim q) \wedge (r \vee \sim r)]$ (commutative)

5. $\sim p \vee [(\sim r \vee \sim q) \wedge \mathbf{t}]$ (negation)
6. $\sim p \vee (\sim r \vee \sim q)$ (identity)

For Problem C:

1. $\sim q \wedge (\sim p \wedge [\sim (p \wedge \sim r) \vee (\sim p \wedge q)])$ (given)
2. $\sim q \wedge (\sim p \wedge [(\sim p \vee \sim (\sim r)) \vee (\sim p \wedge q)])$ (de Morgan's)
3. $\sim q \wedge (\sim p \wedge [(\sim p \vee r) \vee (\sim p \wedge q)])$ (double negative)
4. $\sim q \wedge ([\sim p \wedge (\sim p \vee r)] \vee [\sim p \wedge (\sim p \wedge q)])$ (distributive)
5. $\sim q \wedge (\sim p \vee [\sim p \wedge (\sim p \wedge q)])$ (absorption)
6. $\sim q \wedge (\sim p \vee [(\sim p \wedge \sim p) \wedge q])$ (associative)
7. $\sim q \wedge (\sim p \vee (\sim p \wedge q)])$ (idempotent)
8. $\sim q \wedge \sim p$ (absorption)

Problem 1.3

Show the correctness or falsehood of the following expressions (shown as equivalences) ***without*** *using a truth table, and cite the rule used.*

a. $\sim (p \rightarrow ((q \vee \sim (p \vee q)) \vee (\sim p \wedge q))) \ \equiv \sim (p \rightarrow q) \wedge p$

b. $(p \wedge [(\sim (\sim p \vee q)) \vee (p \wedge r))] \ \equiv \ p$

For Problem A:

1. $\sim (p \rightarrow ((q \vee \sim (p \vee q)) \vee (\sim p \wedge q)))$ (given)
2. $\sim (p \rightarrow ((q \vee (\sim p \wedge \sim q)) \vee (\sim p \wedge q)))$ (de Morgan's)
3. $\sim (p \rightarrow ([q \vee \sim p] \wedge [q \vee \sim q]) \vee (\sim p \wedge q))$ (distributive)
4. $\sim (p \rightarrow ([q \vee \sim p] \wedge \mathbf{t}) \vee (\sim p \wedge q))$ (negation)
5. $\sim (p \rightarrow ([q \vee \sim p] \vee (\sim p \wedge q)))$ (identity)

6. $\sim (p \to ([(q \vee \sim p) \vee \sim p] \wedge [(q \vee \sim p) \vee q]))$ (distributive)
7. $\sim (p \to ([(q \vee \sim p) \vee \sim p] \wedge [q \vee (q \vee \sim p)]))$ (commutative)
8. $\sim (p \to ([q \vee (\sim p \vee \sim p)] \wedge [(q \vee q) \vee \sim p]))$ (associative)
9. $\sim (p \to ([q \vee \sim p] \wedge [q \vee \sim p]))$ (idempotent)
10. $\sim (p \to (q \vee \sim p))$ (idempotent)
11. $\sim (\sim p \vee (q \vee \sim p))$ (definition of $\to$)
12. $\sim (\sim p \vee (\sim p \vee q))$ (commutative)
13. $\sim ((\sim p \vee \sim p) \vee q)$ (associative)
14. $\sim (\sim p \vee q)$ (idempotent)
15. $\sim (\sim p) \wedge \sim q$ (de Morgan's)
16. $p \wedge \sim q$ (double negative)
17. The other side is equivalent to $\sim (\sim p \vee q) \wedge p$ by definition of $\to$, which is equivalent to $p \wedge \sim q \wedge p$ by de Morgan's and double negative, which reduces to $p \wedge \sim q$, so the expressions are logically equivalent.

For Problem B:

1. $(p \wedge [(\sim (\sim p \vee q)) \vee (p \wedge r))] \equiv p$ (given)
2. $p \wedge [(p \wedge \sim q) \vee (p \wedge r)]$ (DeMorgan's Law)
3. $[p \wedge (p \wedge \sim q)] \vee [p \wedge (p \wedge r)]$ (distributive)
4. $[(p \wedge p) \wedge \sim q] \vee [(p \wedge p) \wedge r]$ (associative)
5. $(p \wedge \sim q) \vee (p \wedge r)$ (idempotent)
6. The other side - p - is not logically equivalent to the left expression.

Problem 1.4

Identify the form of the following arguments. If there is an error, identify the ***type of error****.*

a. *Nothing intelligible ever puzzles me. Logic puzzles me. Therefore, logic is unintelligible.*

b. *If the car doesn't start, it is broken. The car is broken. Therefore, the car doesn't start.*

a. Valid modus tollens argument

   The logical syllogism is valid, in the form of modus tollens or contrapositive (or even transitive, in book parlance). The first statement declares $i \rightarrow \sim p$, where i means the subject is intelligible and p is the speaker's puzzled state. (In other words, intelligible things do not puzzle the speaker.) The second statement declares p is true. The last statement means that $\sim i$ (or in full, that $p \rightarrow \sim i$), which is the contrapositive of the first statement (recall: $i \rightarrow \sim p$). This is therefore a valid modus tollens argument.

b. Converse Error

   The car starting is p and it being broken is q. The first two statements imply that $\sim p \rightarrow q$ and q is true. The conclusion states therefore that $q \rightarrow \sim p$, which is unmerited. The conclusion is a converse error.

Problem 1.5

The Mona Lisa was just stolen from the Louvre in Paris! V.I.L.E. henchmen are brought into ACME headquarters for questioning. The chief determines that all V.I.L.E. agents are lying except for one, and is able to figure out which V.I.L.E. henchman stole the Mona Lisa from the following statements:

a. *Vic the Slick: Wonder Rat stole the Mona Lisa.*

b. *Contessa: Top Grunge didn't steal the Mona Lisa.*

c. *Wonder Rat: Top Grunge was in Seattle with Vic the Slick when the Mona Lisa was stolen.*

d. Top Grunge: Wonder Rat didn't steal the Mona Lisa.

Who stole the Mona Lisa and who was telling the truth?

Top Grunge stole the Mona Lisa and was also telling the truth.

We can rewrite the statements as follows where v is Vic the Slick, c is the Contessa, w is Wonder Rat, and t is Top Grunge.

a. w

b. $\sim t$

c. $\sim t \wedge \sim v$

d. $\sim w$

If statement b is false, then statement c is false. If statement c is false, then statement b is false. Thus, statements b and c must both be false, since only one cannot be true. Since statement b is false, then Top Grunge stole the Mona Lisa. Statements a and d cannot both be false. Since we already determined that Top Grunge stole the Mona Lisa, Wonder Rat could not have stolen the Mona Lisa, so statement a is false and statement d is true.

Problem 1.6

Translate each of these statements into logical expressions using predicate quantifiers, and logical connectives. Each statement must be translated in two ways: First, let the domain consist of the students in your class and second, let it consist of all people. Please define your predicate (For example, C(x) could be the predicate: x is in your class)

a. Everyone in your class has a cellular phone.
Let A(n) denote the statement: n has a cellular phone.
Let C(n) denote the statement: n is in your class.
Students: $\forall x, A(x)$
All people: $\forall x, A(x) \wedge C(x)$

b. Somebody in your class has seen a foreign movie.
Let F(n) denote the statement: n has seen a foreign movie.

Let C(n) denote the statement: n is in your class.
Students: $\exists x, F(x)$
All people: $\exists x, F(x) \wedge C(x)$

c. *There is a person in your class who cannot swim.*
Let S(n) denote the statement: n can swim.
Let C(n) denote the statement: n is in your class.
Students: $\exists x, \sim S(x)$
All people: $\exists x, \sim S(x) \wedge C(x)$

d. *All students in your class can solve quadratic equations.*
Let Q(n) denote the statement: n can solve quadradic equations.
Let C(n) denote the statement: n is in your class.
Students: $\forall x, Q(x)$
All people: $\forall x, Q(x) \wedge C(x)$

e. *Some student in your class does not want to be rich.*
Let R(n) denote the statement: n wants to be rich.
Let C(n) denote the statement: n is in your class.
Students: $\exists x, \sim R(x)$
All people: $\exists x, \sim R(x) \wedge C(x)$

The following problem is **non-collaborative**—***discuss it with no one but the professor and the TAs.***

Non-collaborative Problem 1.7

Given the following definitions, translate the given English sentences into the logical notation we've been using in class:

a: "Carmen Sandiego is missing."

b: "The supreme court building is stolen."

c: "V.I.L.E. agents are in jail."

d: "ACME agents are successful."

1. *Carmen Sandiego is missing and the supreme court building is stolen.*
2. *If ACME agents are successful, then Carmen Sandiego is not missing and V.I.L.E. agents are in jail.*

Translate the following formal statements into English sentences:

3. $\sim c \rightarrow a$
4. $(b \land \sim c) \rightarrow \sim d$

1. $a \land b$
2. $d \rightarrow (\sim a \land c)$
3. If V.I.L.E. agents are not in jail, then Carmen Sandiego is missing.
4. If the supreme court building is stolen and V.I.L.E. agents are not in jail, then ACME agents are not successful.

BONUS Problem:

Problem 1.8

In class, we worked some examples of the knights and knaves puzzle (See Example 1.3.16 in the book). There are much more challenging knights and knaves puzzles. For example, see problem 38(d) on page 42, which would make a nice bonus problem, except for the hint in the back of the book.

Bonus: *Construct a suitably challenging knights and knaves puzzle involving three or more individuals.*

Full disclosure, Part 1*. A simple Google search of "knights and knaves" yields this webpage with 382 computer generated Knights and Knaves puzzles:* http://www.hku.hk/cgi-bin/philodep/knight/puzzle. *We trust (honor code) that you won't copy one of these puzzles and claim it is your own creation. However, finding the solution for one of the more challenging puzzles is acceptable.*

Full disclosure, Part 2*. Had you chosen instead to Google "knights and knaves solution", you would find this page:*
http://wiki.soe.ucsc.edu/bin/view/SoeClasses/AIClassKnightsKnaves

with a computer program that solves the knights and knaves puzzles on the webpage above, AND other knights and knaves. We trust (honor code) that you won't use this solver to solve one of the 382 problems and claim the solution as your own. However, using this parser and some ingenuity to find and check a NEW knights and knaves puzzle (not one of the 382) is acceptable.