CSCI 0220 **Discrete Structures and Probability** F. Preparata

Homework 1

Solution Key

All homeworks are due at 1:00pm in the CS22 bin on the CIT second floor, opposite the elevators.

Write your full name *and the problem number on each piece of paper you hand in and then staple.*

As was done in class, please use the "0" and "1" convention for truth values, and use the standard truth table row ordering.

Reading*: Textbook sections* 1.1, 1.2, 1.3 *(required)*, 1.4 *(recommended).*

Problem 1.1

Using a ***truth table****, decide if each of the following expressions is a contradiction, a tautology, or neither.*

a. $[(p \rightarrow r) \vee (r \rightarrow q)] \wedge (p \wedge \sim q)$

b. $[(r \wedge p) \vee \sim r] \vee [p \rightarrow (p \wedge \sim q)]$

c. $[(p \wedge q) \vee (\sim q \wedge r)] \wedge (\sim q \wedge \sim r)$

a. neither

p	q	r	$p \rightarrow r$	$r \rightarrow q$	$p \wedge \sim q$	$(p \rightarrow r) \vee (r \rightarrow q)$	$[(p \rightarrow r) \wedge (r \rightarrow q)] \wedge (p \wedge \sim q)$
T	T	T	T	T	F	T	F
T	T	F	F	T	F	T	F
T	F	T	T	F	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	F	T	F
F	T	F	T	T	F	T	F
F	F	T	T	F	F	T	F
F	F	F	T	T	F	T	F

b. tautology

p	q	r	$p\wedge \sim q$	$r \wedge p$	$p \rightarrow (p\wedge \sim q)$	$(r \wedge p)\vee \sim r$	$[p \rightarrow (p\wedge \sim q)] \vee [(r \wedge p)\vee \sim r]$
T	T	T	F	T	F	T	T
T	T	F	F	F	F	T	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	T	T
F	T	T	F	F	T	F	T
F	T	F	F	F	T	T	T
F	F	T	F	F	T	F	T
F	F	F	F	F	T	T	T

c. contradiction

p	q	r	$p \wedge q$	$\sim q \wedge r$	$\sim q\wedge \sim r$	$(p \wedge q) \vee (\sim q \wedge r)$	$[(p \wedge q) \vee (\sim q \wedge r)] \wedge (\sim q\wedge \sim r)$
T	T	T	T	F	F	T	F
T	T	F	T	F	F	T	F
T	F	T	F	T	F	T	F
T	F	F	F	F	T	F	F
F	T	T	F	F	F	F	F
F	T	F	F	F	F	F	F
F	F	T	F	T	F	T	F
F	F	F	F	F	T	F	F

Problem 1.2

Given the following definitions:

a: *"Vampires don't get along with werewolves."*

b: *"Willy the Werewolf is missing."*

c: *"Vinny the Vampire is pleased."*

d: *"Willy's mother is sad."*

Translate the given English sentences into the logical notation we've been using in class:

1. *Willy the Werewolf is missing and Vinny the Vampire is pleased.*
2. *If Willy the Werewolf is missing, then Willy's mother is sad.*

3. If vampires get along with werewolves and Willy the Werewolf is missing, then Vinny the Vampire is not pleased.

Translate the following formal statements into English sentences:

4. $\sim d \rightarrow \sim b$

5. $(a \wedge \sim c) \rightarrow (\sim b \wedge \sim d)$

6. $b \rightarrow ((a \wedge c) \vee (\sim a \wedge \sim c))$

1. $a \wedge b$
2. $b \rightarrow d$
3. $\sim a \wedge b \rightarrow \sim c$
4. If Willy's mother is not sad, then Willy the Werewolf is not missing.
5. If vampires don't get along with werewolves and Vinny the Vampire is not pleased, then Willy the Werewolf is not missing and Willy's mother is not sad.
6. If Willy the Werewolf is missing, then vampires don't get along with werewolves and Vinny the Vampire is pleased, or Vampires get along with werewolves and Vinny the Vampire is not pleased.

Problem 1.3

Simplify the following expressions using logical equivalences (indicate at each step which equivalence is being used).

a. $((p \vee \sim q) \wedge (\sim p \vee q)) \vee \sim (\sim (p \vee \sim r) \wedge q)$

b. $\sim ((p \vee q) \wedge r) \vee q$

a. $(p \vee \sim q) \wedge (\sim p \vee q) \vee \sim (\sim (p \vee \sim r) \wedge q) \equiv$ (Given)
$((p \vee \sim q) \wedge (\sim p \vee q)) \vee (p \vee \sim r \vee \sim q) \equiv$ (DeMorgan's)
$((p \vee \sim q) \vee (p \vee \sim r \vee \sim q)) \wedge ((\sim p \vee q)) \vee (p \vee \sim r \vee \sim q)) \equiv$ (Distributive Law)
$p \vee \sim q \vee \sim r$ (Idempotent and Universal Bound)

b. $\sim\ ((p \vee q) \wedge r) \vee q \equiv$
$(\sim\ (p \vee q) \vee \sim\ r) \vee q \equiv$ (DeMorgan's)
$(\sim\ p \wedge \sim\ q) \vee \sim\ r \vee q \equiv$ (DeMorgan's)
$((\sim\ p \vee q) \wedge (\sim\ q \vee q)) \vee \sim\ r \equiv$ (Distributive (with q))
$\sim\ p \vee q \vee \sim\ r$ (Universal Bound and Negation)

Problem 1.4

Show the correctness or falsehood of the following equivalences, without using a truth table (indicate at each step which equivalence is being used).

a. $\sim (p \rightarrow ((q \vee \sim (p \vee q)) \wedge (q \rightarrow p))) \ \equiv\ p \wedge \sim q$

b. $(p \wedge ((\sim (\sim p \vee q)) \vee (p \wedge r))) \ \equiv\ p$

a.

$$
\begin{array}{ll}
cl & \sim (p \rightarrow ((q \vee \sim (p \vee q)) \wedge (q \rightarrow p))) \\
\equiv & \sim (p \rightarrow ((q \vee (\sim p \wedge \sim q)) \wedge (q \rightarrow p))) \\
\equiv & \sim (p \rightarrow (((q \vee \sim p) \wedge (q \vee \sim q)) \wedge (q \rightarrow p))) \\
\equiv & \sim (p \rightarrow (((q \vee \sim p) \wedge t) \wedge (q \rightarrow p))) \\
\equiv & \sim (p \rightarrow ((q \vee \sim p) \wedge (q \rightarrow p))) \\
\equiv & \sim (\sim p \vee ((q \vee \sim p) \wedge (\sim q \vee p))) \equiv \qquad p \wedge \sim ((q \vee \sim p) \wedge (\sim q \vee p)) \\
\equiv & p \wedge (\sim (q \vee \sim p) \vee \sim (\sim q \vee p)) \\
\equiv & p \wedge ((\sim q \wedge p) \vee (q \wedge \sim p)) \\
\equiv & (p \wedge (\sim q \wedge p)) \vee (p \wedge (q \wedge \sim p)) \\
\equiv & (p \wedge (p \wedge \sim q)) \vee (p \wedge (\sim p \wedge q)) \\
\equiv & ((p \wedge p) \wedge \sim q) \vee ((p \wedge \sim p) \wedge q)) \\
\equiv & (p \wedge \sim q) \vee (c \wedge q) \\
\equiv & (p \wedge \sim q) \vee c \\
\equiv & (p \wedge \sim q)
\end{array}
$$

Therefore, the two statements are equivalent.

b.

$$
\begin{array}{cc}
cl & p \wedge ((\sim (\sim p \vee q)) \vee (p \wedge r)) \\
\equiv & p \wedge ((p \wedge \sim q) \vee (p \wedge r)) \\
\equiv & p \wedge (p \wedge (\sim q \vee r)) \\
\equiv & (p \wedge p) \wedge (\sim q \vee r)) \\
\equiv & p \wedge (\sim q \vee r)
\end{array}
$$

This statement is not equivalent to p, because it can be false even when p is true. (For instance, if q is true or r is false.)

Problem 1.5

Give the contrapositive, converse, and inverse of each of the following statements.

a. *If Catherine is going to the Providence Place Mall, then she is accompanied by Thea and Chris.*

   (a) Contrapositive: If Catherine is not accompanied by Thea and Chris, then she is not going to the mall.

   (b) Converse: If Catherine is accompanied by Thea and Chris, then she is going to the Providence Place Mall.

   (c) Inverse: If Catherine is not going to the Providence Place Mall, then she is not accompanied by Thea and Chris.

b. *If Alex does not sit down in the SciLi before 10 PM, then he will not complete his homework on time.*

   (a) Contrapositive: If Alex does complete his homework on time, then he sat down in the SciLi before 10 PM.

   (b) Converse: If Alex does not complete his homework on time, then he did not sit down in the SciLi before 10 PM.

   (c) Inverse: If Alex does sit down in the SciLi before 10 PM, then he will complete his homework on time.

c. *If Ben plans on cleaning his dorm room, then he plans on asking Greg for garbage bags and does not plan to invite Tom over to watch soccer.*

(a) Contrapositive: If Ben does not plan on asking Gregt for garbage bags or does plan to invite Tom over to watch soccer, then he does not plan on cleaning his dorm room.

(b) Converse: If Ben plans on asking Greg for garbage bags and does not plan to invite Tom over to watch soccer, then he plans on cleaning his dorm room.

(c) Inverse: If Ben does not plan on cleaning his dorm room, then he does not plan on asking Greg for garbage bags or does plan to invite Tom over to watch soccer.

Problem 1.6

*Identify the form of the following arguments. If there is an error, identify the **type of error**.*

a. *Nothing intelligible ever puzzles me. Logic puzzles me. Therefore, logic is unintelligible.*

b. *If the car doesn't start, it is broken. The car is broken. Therefore, the car doesn't start.*

a. Valid modus tollens argument

The logical syllogism is valid, in the form of modus tollens or contrapositive (or even transitive, in book parlance). The first statement declares $i \to \sim p$, where i means the subject is intelligible and p is the speaker's puzzled state. (In other words, intelligible things do not puzzle the speaker.) The second statement declares p is true. The last statement means that $\sim i$ (or in full, that $p \to \sim i$), which is the contrapositive of the first statement (recall: $i \to \sim p$). This is therefore a valid modus tollens argument.

b. Converse Error

The car starting is p and it being broken is q. The first two statements imply that $\sim p \to q$ and q is true. The conclusion states therefore that $q \to \sim p$, which is unmerited. The conclusion is a converse error.