**CSCI 0220 Discrete Structures and Probability** F. Preparata

# Homework 3

*Due: 20 Feb 2009*

All homeworks are due at 1:00pm in the CS22 bin on the CIT second floor, opposite the elevators.

Write your *full name* and the problem number on each piece of paper you hand in and then staple.

**Reading**: Chapter 3: Sections 3.6-3.7. Chapter 4: Section 4.1-4.4 (except material after pg. 241).

### Problem 3.1

For all integers $n \geq 0$, prove $n^3 + (n+1)^3 + (n+2)^3$ is divisible by 9.

### Problem 3.2

Prove that $\sqrt[3]{3}$ is irrational. (You may assume the following Lemma: "For any integer $n$, if $n^3$ is divisible by 3, then $n$ is divisible by 3.")

### Problem 3.3

Prove that $\forall n \in \mathbb{Z},\ n^2 + 3n - 5$ is odd using division of cases.

### Problem 3.4

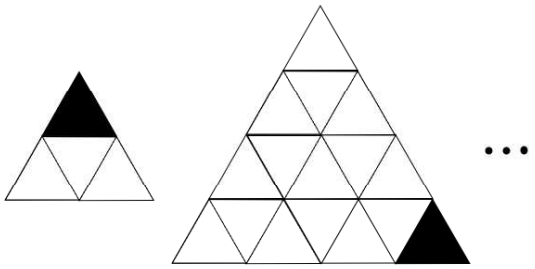
Using the well-ordering principle, prove that every integer $n$ greater than 1 is either a prime number or a product of prime numbers.

### Problem 3.5

Given any integers $a, b, c$, if $a - b$ is odd and $b - c$ is even, what can you say about the parity (odd/even) of $a - c$? Support your answer with a proof.

## Problem 3.6

Consider the set of equilateral triangles with sides of length $2^n$, where $n > 0$, with one triangle $\triangle$ of side length 1 missing from *any* one of the three corners of the larger triangle (see picture below).



Show that each figure in the set can be covered with tiles composed of three side-length-1 triangles $\triangle$ in the form: