CSCI 0220 Discrete Structures and Probability F. Preparata

Homework 4

Due: 27 Feb 2009

All homeworks are due at 1:00pm in the CS22 bin on the CIT second floor, opposite the elevators.

Write your *full name* and the problem number on each piece of paper you hand in and then staple.

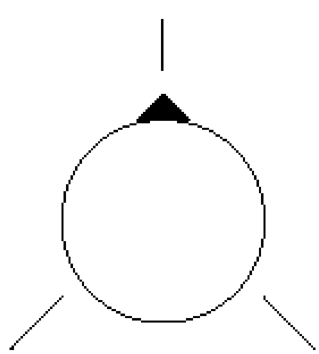
Reading: Chapter 3: Section 3.8. Chapter 4: Section 4.5 pp. 619-620.

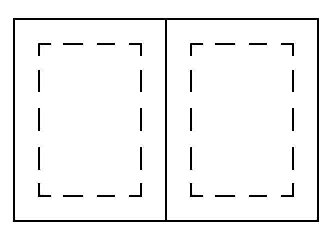
Problem 4.1

Your werewolf friend Horowitz recently acquired a strange artifact from a vampire's clutches. It looks a little bit like a safe. On the front of the safe is a dial that can be turned to three valid positions. The dial is connected to a large digital display that has the amazing property of being able to display arbitrarily large non-negative integers.

The dial starts in the upward position, and each time the dial is turned one "click" *in either direction*, the display's inner count increases by one (it starts at "0"). However, the display only shows the count when the dial is in the upward position. This means, for example, that it is imposible to ever get the number "1" to display, since the dial must end pointing up.

You can turn the dial in either direction as much as you want. Every time it goes back to its original position (the upward position), the display shows how many clicks happened.





It is easy to see that we can make the number "3" appear on the display:

we simply turn the dial three clicks in one direction. As we've already said, though, it's impossible to make the number "1" appear.

Horowitz wants to know whether all other positive integers can be made to appear on the display. He also wants you to justify your answer.

Note: The numbers only increase; while you can turn the dial in either direction, both directions just increase the number, and there is no way to make the numbers decrease. Remember that the display can hold an arbitrarily large integer.

Problem 4.2

If you have \$3-bills and a \$5-bills you can express any amount over \$7. Using induction, prove this theorem.

Problem 4.3

Prove by induction that

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Problem 4.4

Suppose that $h_0, h_1, h_2, h_3, \ldots$ is a sequence defined as follows: $h_0 = 1, h_1, = 2, h_2 = 3, h_k = h_{k-1} + h_{k-2} + h_{k-3}$ for all integers $k \geq 3$. Prove that $h_n \leq 3^n$ for all integers $n \geq 0$.

Problem 4.5

The game Mini-nim is defined as follows: Some positive number of sticks are placed on the ground. Two players take turns removing one, two, or three sticks. The player to remove the last one loses. Use the second form of induction to show that the second player has a winning strategy if and only if the number of remaining sticks n equals $4k + 1$ for some $k \in \mathbb{Z}$.