

Homework 5

Due: 6 Mar 2009

All homeworks are due at 1:00pm in the CS22 bin on the CIT second floor, opposite the elevators.

Write your *full name* and the problem number on each piece of paper you hand in and then staple.

Reading: Chapter 5 (up to pg. 294, excluding "Halting Problem").

Problem 5.1

Let A, B, C be sets, and assume they are all subsets of a universal set U . Using the **element method**, show that the following equivalences hold:

- a. $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$
- b. $A \cup (B \cap (C - A)) = A \cup (B \cap C)$
- c. $(A - B) \cap (B - C) \cap (A - C) = \emptyset$

Problem 5.2

Let A, B, C be sets, and assume they are all subsets of a universal set U . Using **set algebra**, show that the following equivalences hold:

- a. $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$
- b. $A \cup (B \cap (C - A)) = A \cup (B \cap C)$
- c. $(A - B) \cap (B - C) \cap (A - C) = \emptyset$

Problem 5.3

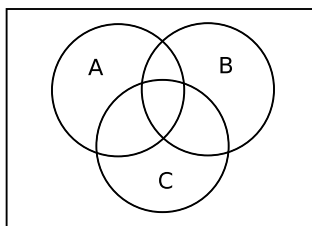
Prove or disprove that $X \cap Z \subseteq Y \Leftrightarrow (X - Y) \cup (Y - Z) \subseteq Z^c$

Problem 5.4

Let $A, B, C \subseteq U$. Prove or disprove that $(A-B) \subseteq C$ if and only if $(A-C) \subseteq B$.

Problem 5.5

Shade the area of the following Venn diagram which corresponds to each given set (for each case, produce the corresponding instance of the Venn diagram):



- a. $(A - B) \cup C$
- b. $(A \cap B) - C$
- c. $((A \cup C) \cup (B \cup C)) \cap (A \cap B)^c$

Problem 5.6

Given sets:

$$A = \{1, 4, 6, 7, 12\}$$

$$B = \{2, 4, 6, 8, 10, 12\}$$

$$C = \{3, 4, 6, 8, 12\}$$

Give:

- a. The list of elements in $(A \cap B^c) \cap C^c$
- b. The cardinality of the power set of $(B \cup C) \cap A^c$
- c. The list of members of the power set of $(A \cap B) \cap C$