CSCI 0220 **Discrete Structures and Probability** F. Preparata

Homework 7

Due: 3 Apr 2009

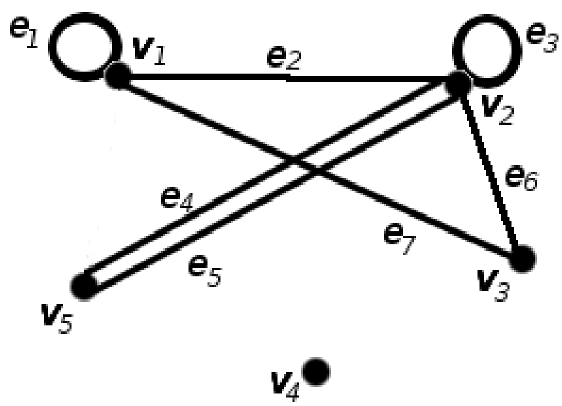
All homeworks are due at 1:00pm in the CS22 bin on the CIT second floor, opposite the elevators.

Write your *full name* and the problem number on each piece of paper you hand in and then staple.

Reading: Chapter 11: 11.1-11.6 (up to p. 728); Chapter 7, Section 7.1-7.2 (up to p. 406)

Problem 7.1

In the following graph, vertices are denoted by v's and edges by e's. Do the following:



a) Find all edges that are incident on v_1

b) Find all vertices that are adjacent to v_3

c) Find all edges that are adjacent to e_1

d) Find all loops

e) Find all parallel edges

f) Find all isolated vertices

g) Find the degree of v_3

h) Find the total degree of the graph

Problem 7.2

For each of a-d), either draw a graph with the specified properties or explain why no such graph exists:

a) Graph with four vertices of degrees 1, 1, 1, and 4.

b) Graph with four vertices of degrees 1, 2, 3, and 4.

c) Simple graph with five vertices of degrees 1, 1, 1, 2, and 3.

d) Simple graph with four vertices of degrees 1, 2, 3, and 4.

Problem 7.3

Let G be a connected graph and let C be a circuit in G. Let G' be the subgraph obtained by removing all the edges of C from G and also any vertices that become isolated when the edges of C are removed. Prove that if G' is nonempty, then there exists a vertex v such that v is in both C and G'. The following lemma will be helpful: If a graph G is connected, then any two distinct vertices of G can be connected by a simple path.

Problem 7.4

Prove that given any two distinct vertices of a tree, there exists a unique path from one to the other.

Problem 7.5

Let $G = (V, E)$ be a simple graph. Let R be the relation on V consisting of pairs of vertices (u, v) such that there is a path from u to v or such that $u = v$. Show that R is an equivalence relation.

Problem 7.6

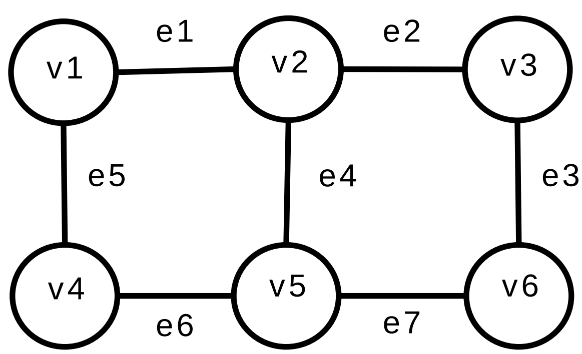
The Brown CS department has decided to secede from Facebook, creating their own internal social network called CodeBook. For space considerations, the programmers need to know how many possible friendships will exist in this new network. Determine the $m$aximum total number of friendships that can exist in CodeBook if there are n people in the network, and prove your solution.

Problem 7.7

Show that in any simple graph $G = (V, E)$ there is a path from any vertex v of odd degree to some other vertex of odd degree.

Problem 7.8

Consider the graph G shown below:



How many spanning trees does it have?