Homework 8

Due: 10 Apr 2009

All homeworks are due at 1:00pm in the CS22 bin on the CIT second floor, opposite the elevators.

Write your *full name* and the problem number on each piece of paper you hand in and then staple.

Reading: Chapter 6: Section 6.1 and 6.2. Chapter 7: Section 7.2 (pp. 415-417), 7.3, 7.4, 7.5.

Problem 8.1

For $\Sigma = \{0, 1\}$, draw finite state machines that accept:

a. All strings that contain the subsequence 101.

   Note: A subsequence is an ordered selection of characters from a string. For example, the strings 1001 *and* 000100010 *contain the sequence* 101 *as a subsequece.*

b. All strings that contain the substring 101.

   Note: Characters in substrings must be consecutive. So the strings 1001 *and* 000100010 *do NOT contain the substring* 101*, but* 0110010100 *does.*

c. All strings that end with 101.

Problem 8.2

Construct the state diagram of a finite state machine that when given the sequence of the digits of a number (in base 10) from right to left, computes the remainder of its division by 9. Will the machine still work if given the digits of the number in the opposite order? Be sure to justify your answer and briefly explain how your machine works.

Problem 8.3

Explain why f is one-to-one (injection), onto (surjection), or a bijection. Show EACH attribute by proof or counterexample where appropriate.

a. $f\colon \mathbb{R} \to \mathbb{R}$: $f(x) = e^x$

b. $f\colon \mathbb{R} \to \mathbb{R}^+$: $f(x) = x^4$

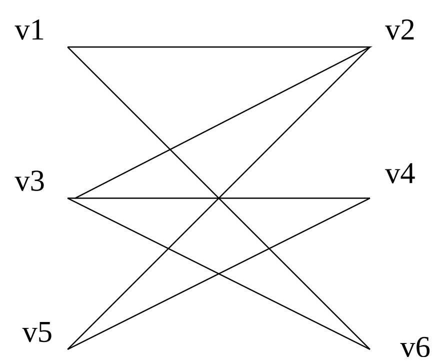
c. $f\colon \mathbb{N} \to \mathbb{N}$: $f(x) = 2x$

d. $f\colon \mathbb{R} \to \mathbb{Z}$: $f(x) = \lceil x \rceil$

Problem 8.4

A *2-colored graph* G is a simple graph each vertex of which can be assigned one of two colors so that no edge connects two vertices of the same color.

For example, the graph below is 2-colored:



Prove that an undirected 2-colored graph cannot contain a cycle that has an odd number of vertices.

Problem 8.5

Let $S = \{1, 2, \ldots, 2n\}$ for some integer n. Show that for any $T \subset S$ such that $|T| = n+1$, there are elements $x, y \in T$ such that x and y are relatively prime.

(Hint: Show that there are elements $x, y \in T$ such that $|x - y| = 1$, then go from there.)

Problem 8.6

Using composability of functions show:

a. $f(y) = e^{3y^2+2} + 1$ is not injective from $\mathbb{R}$ to $\mathbb{R}$

b. $f(x) = (2x+4)^5$ is bijective from $\mathbb{R}$ to $\mathbb{R}$