

## Homework 9

*Due: 17 Apr 2009*

All homeworks are due at 1:00pm in the CS22 bin on the CIT second floor, opposite the elevators.

Write your *full name* and the problem number on each piece of paper you hand in and then staple.

**Reading:** Chapter 6: 6.3, 6.4, 6.5, 6.6, 6.7.

**Note:** Please show your work and explain your reasoning on each problem in this assignment. We can not award partial credit to students for incorrect answers that do not show their work, neither can we give full credit to students for correct answers that do not show their work.

### Problem 9.1

The CS22 TAs meet every Wednesday for a staff meeting at a circular table. Each meeting is attended by 5 male individuals (Alex, Ashwin, Ben, Chris, and Tom) and three women (Nell, Catherine, and Thea).

- a. Alex decided that he wants to sit between two women. He removed one of the chairs so now the round table has 7 chairs. The women arrived first and, as before, sat next to each other. Then the guys arrived and occupied the four remaining seats. Finally, Alex arrived and brought an extra chair, which he placed so as to sit between two women. How many possible arrangements of the TAs at the round table are there in this case? ALSO, for the general case, what if we have  $w$  women and  $m$  men (including Alex)?
- b. In how many ways can you sit the men and women around the circular table so that no two women sit next to each other?
- c. Nell decided to treat the TAs to chocolate. There are five flavored chocolates (Amaretto, Hazelnut, Almond, Fish, and Orange) and three plain chocolates (White, Dark and Milk). The TAs will have one chocolate each. The men will have the flavored chocolates, while the women will have the plain chocolates. How many ways are there to distribute chocolates to TAs? What if there are  $w$  women,  $m$  men,

$w + m$  distinct chocolates,  $w$  of which are meant for women, and the remaining  $m$  meant for men? Is there a bijection to the set of arrangements for part (a)?

- d. The next time around, Nell left eight Milk chocolates for the TAs to eat during the grading session. The men arrived first and decided that (1) each man should have at least one chocolate; and (2) all chocolate should be distributed to men. How many ways are there to distribute these chocolates to the TAs, if the men get their way? (Note that all chocolates are the same, the only thing that matters is how many chocolates each TA got to eat.) What if we have  $m$  men and  $c \geq m$  pieces of chocolate?

### Problem 9.2

Consider a standard deck of 52 cards. How many 5-card sets contain

- the queen of hearts?
- no cards with values lower than 5 (aces have value 1)?
- exactly 1 pair?
- at least 3 diamonds?

### Problem 9.3

Give the coefficient of the given term in the expanded expression

- $x^4y^7$  in  $(3x^2 + 5y)^9$
- $p^{12}q^{44}$  in  $(7p^3 + 2q^4)^{15}$

**Problem 9.4**

How many integer solutions are there to the system

$$x_1 + x_2 + x_3 + x_4 = 40$$

$$1 \leq x_1 \leq 5$$

$$2 \leq x_2 \leq 7$$

$$3 \leq x_3 \leq 9$$

$$5 \leq x_4$$

**Problem 9.5**

Seven people enter an elevator in the basement. Each exists at floor 1,2,3, or 4. In how many ways can this happen? Explain.

**Extra Credit****Problem 9.6**

Use the inclusion-exclusion principle to find the number of primes not exceeding 100.