CSCI 0220 **Discrete Structures and Probability** F. Preparata

Homework 10

Due: 17 Apr 2009

All homeworks are due at 1:00pm in the CS22 bin on the CIT second floor, opposite the elevators.

Write your *full name* and the problem number on each piece of paper you hand in and then staple.

Reading: Chapter 8: 8.1, 8.2, 8.3. Class notes chapters 1, 2.

Note: Please show your work and explain your reasoning on each problem in this assignment. We cannot award partial credit to students for incorrect answers that do not show their work, nor can we give full credit to students for correct answers that do not show their work.

Problem 10.1

a. A vampire arrives in Boston at day 0 and starts biting people at day 1. People bitten become vampires themselves. New vampires bite two persons on the next day after they were bitten and six persons every day afterwards. Write the recurrence relation of the number of vampires in n days for $n \geq 2$, assuming the first vampire is a newly made vampire.

b. Solve the recurrence using the characteristic equation.

c. Prove that your answer satisfies the recurrence relation.

Problem 10.2

In how many ways can a 2 x n rectangle be tiled by 1 x 2 blocks? Prove your answer.

Problem 10.3

Suppose you have a basket containing 12 blue blocks and 6 white blocks. You draw 4 blocks from the basket without replacement. Assume all blocks are equally likely to be selected, and blocks of the same color are indistinguishable.

a. List the simple events in the sample space. Order does not matter.

b. List the simple events for which there are exactly 2 blue blocks.

Now suppose you add 2 yellow blocks and 3 red blocks to the 18 blocks already in the basket. Again, draw 4 blocks without replacement.

c. What is the probability that all 4 blocks will be the same color?

d. What is the probability that all 4 blocks will be different colors, given that at least one block is yellow?

Problem 10.4

Are the following events independent?

a. When rolling a die, that an even number shows up and that a number greater than three shows up.

b. When flipping a coin twice, that the first flip is a heads and that the two flips match.

c. When flipping a coin three times, the first coin flipping comes up tails and two (but not three) heads come up in a row.

Problem 10.5

You have a 100-inch glass ruler, with notches at each inch point from 0 to 100, inclusive. You drop it. It breaks at some location $k + \frac{1}{2}$, where k is an integer from 0 to 99. Each k is equally likely.

a. What's the expected number of "inch marks" on the shorter piece?

   To make this clearer, if we were talking about a 10-inch ruler and it broke at 7.5, there would be marks at 0, 1 2, 3, 4, 5, 6, and 7 on the longer piece, and marks at 8, 9, and 10 on the shorter piece, so the "number of marks" on the smaller piece would be three.

b. What about the long end of the stick?