Notes on Induction

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In correcting the homeworks, we have noticed several commonly recurring problems with proofs by induction. This list may be added to as the semester progresses.

Base Cases

Check the base case!

We have been taking off $\frac{1}{3}$ of all the available points on problems for simply not including the base case. These are essentially free points for you, but proving the base case is absolutely crucial to having a valid proof.

Sometimes the base case is really base cases.

Consider the following proof.
Theorem. *In a group of n people, everybody has the same name.*

Proof. We will proceed via induction. Consider the case $n = 1$. This is trivially true. Now consider a group of size n. We will refer to this group as G. Let $A, B, C \in G$. By the inductive hypothesis, $G - \{A\}$ must be a group of all the same name. Similarly, $G - \{B\}$ must also be a group of all the same name. Since $B, C \in G - \{A\}$ and $A, C \in G - \{B\}$, we have that the name of A equals the name of B equals the name of C equals the name of everybody else. □

Obviously this statement is false; we checked $n = 1$, but failed to check $n = 2$. The statement is (usually) false for $n = 2$, so the inductive chain of logical arguments fails.

True for $n = 1, 2, 3$ does not imply true for all n (nor does true for $n = 1, 2, 3, 4$).

Consider the following proof.
Theorem. *For $n \geq 0 \in \mathbb{Z}$, there exists 4 prime numbers between* $10n + 10(n + 1)$.

Proof. Consider $n = 0$. Between 0 and 10, the prime numbers are $2, 3, 5, 7$. This is 4. Now consider $n = 1$. Between 10 and 20, the prime numbers are $11, 13, 17, 19$. This is also 4. Thus, the statement is true. $\square$

Obviously this is false; consider $n = 2$.