CSCI 0220 **Discrete Structures and Probability** B. Raphael

Midterm Practice

TBA

Problem .1

a. Simplify the following:

$$k(x, y, z) = (p(x) \wedge q(y)) \rightarrow \sim r(z)$$

b. Using your solution from part a. rewrite the following statement without $k(x, y, z)$ and with no negated quantifiers.
Note: If you answer part a. incorrectly, but use your answer correctly, you will receive full credit for part b.

$$\sim \forall x \exists y (\sim \exists z (k(x, y, z)))$$

Problem .2

Prove that if $4 \mid n - 3$, then $8 \mid n^2 - 1$.

Problem .3

Prove by induction that $n^3 - n$ is divisible by three for all $n > 0$.

Problem .4

Define a relation R on the set of all real numbers, $\mathbb{R}$, as follows: $\forall x, y \in \mathbb{R}$,

$$xRy \Leftrightarrow x^2 \leq y^2$$

Is R a partial order relation? Prove or give a counterexample.

Problem .5

Consider the sets:
$A = \{1, 2, 3, 5, 8, 13\}$
$B = \{2, 3, 5, 7, 11, 13\}$
$C = \{1, 3, 6, 9, 12\}$

a. Give the set $(B - A) \cap C^c$

b. Give $|\mathscr{P}(A \cap B)|$

Problem .6

Suppose that $h_0, h_1, h_2, h_3, \ldots$ is a sequence defined as follows: $h_0 = 1, h_1, = 2, h_2 = 3, h_k = h_{k-1} + h_{k-2} + h_{k-3}$ for all integers $k \geq 3$. Prove that $h_n \leq 3^n$ for all integers $n \geq 0$.

Problem .7

Let F be the relation defined on $\mathbb{Z}$ by as follows: For all $m, n \in \mathbb{Z}$, $mFn \iff 4 \mid (m - n)$.

a) Prove that F is an equivalence relation.
b) Describe the equivalence classes of F.