Recitation 1

Date: 22 Feb 2009

Problem 1.1

Simplify each expression and cite the **specific rule** used. Verify your simplification using a truth table.

a. $(p \rightarrow \sim q) \wedge (\sim q \vee \sim p)$

b. $\sim (p \wedge r) \vee (\sim q \wedge r)$

c. $\sim q \wedge (\sim p \wedge [\sim (p \wedge \sim r) \vee (\sim p \wedge q)])$

Problem 1.2

Give the contrapositive, converse, and inverse of the following statements.

a. $\forall d \in \mathbb{Z}, \frac{6}{d} \in \mathbb{Z} \rightarrow d = 3$.

b. $\forall n \in \mathbb{Z}$, if n is prime, then $n = 2$ or n is odd.

c. If the square of an integer is odd, then the integer is odd.

Problem 1.3

Reorder the given premises and use contraposition to show that the conclusion follows from the premises.[1]

1. When I work a logic example without grumbling, you may be sure it is one I understand.
2. The arguments in these examples are not arranged in regular order like the ones I am used to.
3. No easy examples make my head ache.

[1] Adapted from Lewis Carroll, *Symbolic Logic* (New York: Dover, 1958), p. 123.

4. I can't understand examples if the arguments are not arranged in regular order like the ones I am used to.

5. I never grumble at an example unless it gives me a headache.

$\therefore$ These examples are not easy.

Problem 1.4

Prove the following statement by contradiction:

For all real numbers x and y, if x is irrational and y is rational then $x - y$ is irrational.

Problem 1.5

Prove the following using induction:

$6 \mid n(n^2 + 5), \forall n \geq 1$