Recitation 2

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Problem 2.1

Suppose that $s_0, s_1, s_2, \ldots$ is a sequence defined as follows

$$\begin{aligned} s_0 &= 12 \\ s_1 &= 29 \\ s_k &= 5(s_{k-1}) - 6(s_{k-2}) \end{aligned}$$

where $k \geq 2$.

Prove that, for $n \geq 0$, $s_n = 5(3^n) + 7(2^n)$.

Problem 2.2

Using the element method, show that

$$A \cup (A \cap B) = A$$

.

Problem 2.3

Using induction, prove that

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \forall n \geq 0$$

Problem 2.4

Prove that

$$(A - B) \cap (A \cap B) = \emptyset$$

a. ...using the element method.

b. ...using algebraic techniques.

Problem 2.5

Use induction to show that $4|3^{2k-1}+1\ \forall k \geq 1$.

Problem 2.6

As computer scientists we have become very familiar with binary (base-2) representations of integers. In fact, it has been shown that unique binary representations exist for all nonnegative integers. More formally, given any positive integer n, n has a unique representation of the form

$$n = c_r \cdot 2^r + c_{r-1} \cdot 2^{r-1} + \ldots + c_2 \cdot 2^2 + c_1 \cdot 2 + c_0$$

where r is a nonnegative integer, $c_r = 1$, and $c_j \in \{0,1\}$ for all $0 \leq j \leq r-1$.

Now consider an alternative, base-3 representation. Prove that all integers $n \geq 1$ can be written in base-3 notation:

$$n = c_r \cdot 3^r + c_{r-1} \cdot 3^{r-1} + \ldots + c_2 \cdot 3^2 + c_1 \cdot 3 + c_0$$

where r is a nonnegative integer, $c_r \in \{1,2\}$, and $c_i \in \{0,1,2\}$ for all $0 \leq i \leq r-1$. (Hint: you need not use strong induction, but it might be worth considering.)