

## Recitation 3

*Date: 22 Apr 2009*

### Problem 3.1

Consider the following recurrence relation:

$$a_{n+1} = a_n + 2a_{n-1}$$

Given the initial conditions  $a_0 = 1$  and  $a_1 = 2$ , solve the recurrence relation by

- a. iteration (prove that your guess is correct)
- b. the characteristic equation

Now consider  $a_{n+1} = a_n + 2a_{n-1} + 10$ . This is known as an inhomogeneous recurrence relation, since there is a term not involving elements of the sequence. However, it can be rewritten as a linear homogeneous recurrence relation (what we have covered in class)!

- c. Consider  $a_{n+2} - a_{n+1}$ . Write a linear recurrence relation based on the simplification of that expression.
- d. Write the characteristic equation and find the roots. How do the roots relate to the homogeneous (original) case?

### Problem 3.2

We have a rack of the 15 balls used in a typical game of billiards. Each ball has a number from 1-15, the first 8 balls are solid colors, the second 7 balls are striped.

- a. How many ways can you select 4 solid-colored balls and 4 striped balls? (Remember that each ball has a unique number...)
- b. How many ways can you select a group of (at least one) prime-numbered balls?

**Problem 3.3**

Among 20 students in a room, 7 study applied mathematics, 10 study economics, and 10 study computer science. Also, 3 study applied mathematics and economics, 4 study applied mathematics and computer science, and 6 study economics and computer science. We know 2 students study all three subjects. How many of the 20 students in the room study none of the three subjects?

**Problem 3.4**

Carmen Sandiego is sending an encrypted message back describing each attribute of Wonder Rat, a treacherous villain. Since each attribute has three choices, he represents Wonder Rat's profile as a ternary string. A ternary string is a finite string composed over the alphabet  $\{a, b, c\}$ . How many ternary strings of length 9 have at least two a's, one b, and four c's? Please calculate the numerical result.