Homework 1

Due: 23 Sep 2009 1:55pm

All homeworks are due at 1:55pm in the **CS 31 bin** on the second floor. No late homeworks are accepted.

Please include your *login name* on each piece of paper you hand in, and please staple your pages together before handing in.

Problem 1.1

Convert the following numbers from hexadecimal to binary to octal without multiplying or dividing. Show the steps you used to arrive at the answer and explain[1] the purpose of each.

(Hint: Think about groupings.)

a. $\texttt{C45A0}_{16}$

b. $\texttt{FF7B2}_{16}$

Problem 1.2

Convert the following octal numbers to binary to hexadecimal. (Again, show your steps and explain them.)

a. $\texttt{42773}_{8}$

b. $\texttt{65413573210}_{8}$

Problem 1.3

Do the following unsigned addition/subtraction problems. Show your carries/borrows! For the purpose of this problem, assume that words have an infinite width. That is, don't truncate.
Remember all work should be written in the base that the problem is in, including carries/borrows.

[1] When we say "explain", a sentence or two is sufficient.

a. $\texttt{ABBABA}_{16} + \texttt{981CD}_{16}$

b. $\texttt{01010111}_2 + \texttt{10111011}_2$

c. $\texttt{73237}_8 - \texttt{55451}_8$

d. $\texttt{11100111111011}_2 - \texttt{10101000111001}_2$

Problem 1.4

Compute the following with 8-bit 2's complement arithmetic, answering the following questions for each problem.

(1) What is the minuend?

(2) What is the subtrahend?

(3) What is the 1's complement of the subtrahend?

(4) What is the 2's complement of the subtrahend?

(5) What is the 2's complement arithmetic sum?

a. *Example:* $(-\texttt{27})_{10} + \texttt{101}_{10}$

   (1) $\texttt{1100101}_2$
   (2) $-\texttt{11011}_2$
   (3) $\texttt{11100100}_2$
   (4) $\texttt{11100101}_2$
   (5) $\texttt{01001010}_2$

b. $(-\texttt{76})_{10} + \texttt{65}_{10}$

c. $(-\texttt{20})_{10} + \texttt{20}_{10}$

Problem 1.5

a. Do the following in 8-bit 2's complement binary arithmetic: $(-\texttt{89}) + (-\texttt{41})$. Be sure to show the stages you went through to get your answer.

b. Explain the problem with this answer and give a simple procedure for detecting it.

Problem 1.6

Recall that the binary two's complement representation generalizes to other bases as the radix complement. There are two ways of calculating the radix complement of a number. Calculate the radix complement of $\texttt{3A3EFCF0}_{16}$ using each of these methods, showing your steps and explaining your work.

Problem 1.7

Older mainframe computers used an encoding for integers called *Binary Coded Decimal (BCD)* that is a compromise betweeen a binary representation and a decimal one. A number is represented in base 10 with each decimal digit represented by four binary digits. For example, the decimal number 7290 is represented as follows:

$$\underbrace{0111}_{7}\underbrace{0010}_{2}\underbrace{1001}_{9}\underbrace{0000}_{0}.$$

Note that there could be a negative number notation for BCD, but for now we are specifically talking about unsigned notation.

Draco is trying to hack some older mainframes, but he can't figure out if he needs to learn BCD or not.

How many bits would be required to code the following numbers in BCD? In binary?

a. 3

b. 1415

c. 92653

Lucius has a suggestion for Draco. He suggests keeping the binary coding, but instead of representing the number in base 10, it should represented in base 13. So, under Lucius's new system, 7290 would be represented as:

$$\underbrace{0011}_{3}\underbrace{0100}_{4}\underbrace{0001}_{1}\underbrace{1010}_{10}.$$

Lucius claims his system, one of pure-blood pedigree, is more efficient than BCD. Is he right? Explain why or why not.