Representation

CS31

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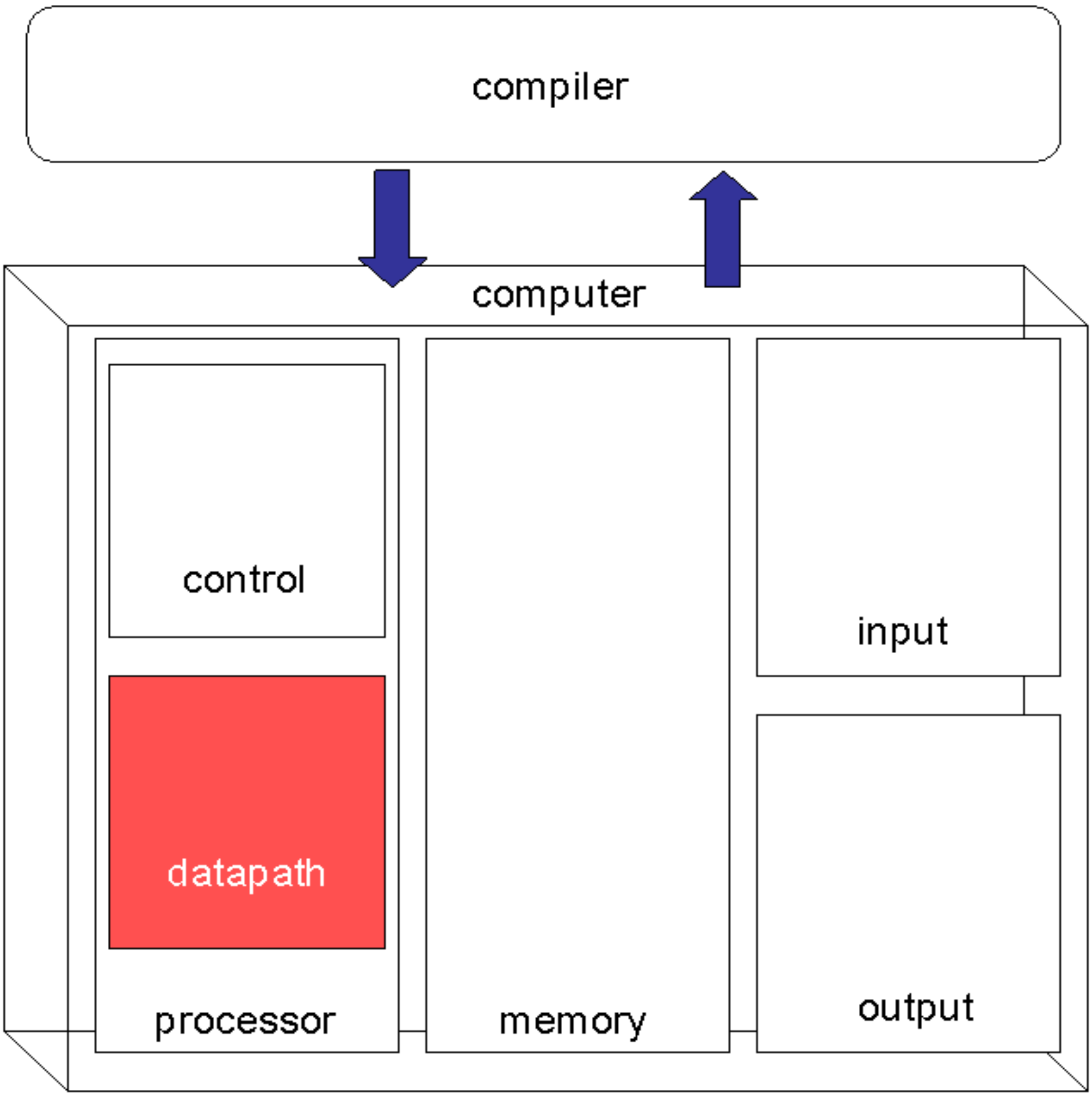
BROWN

Overview

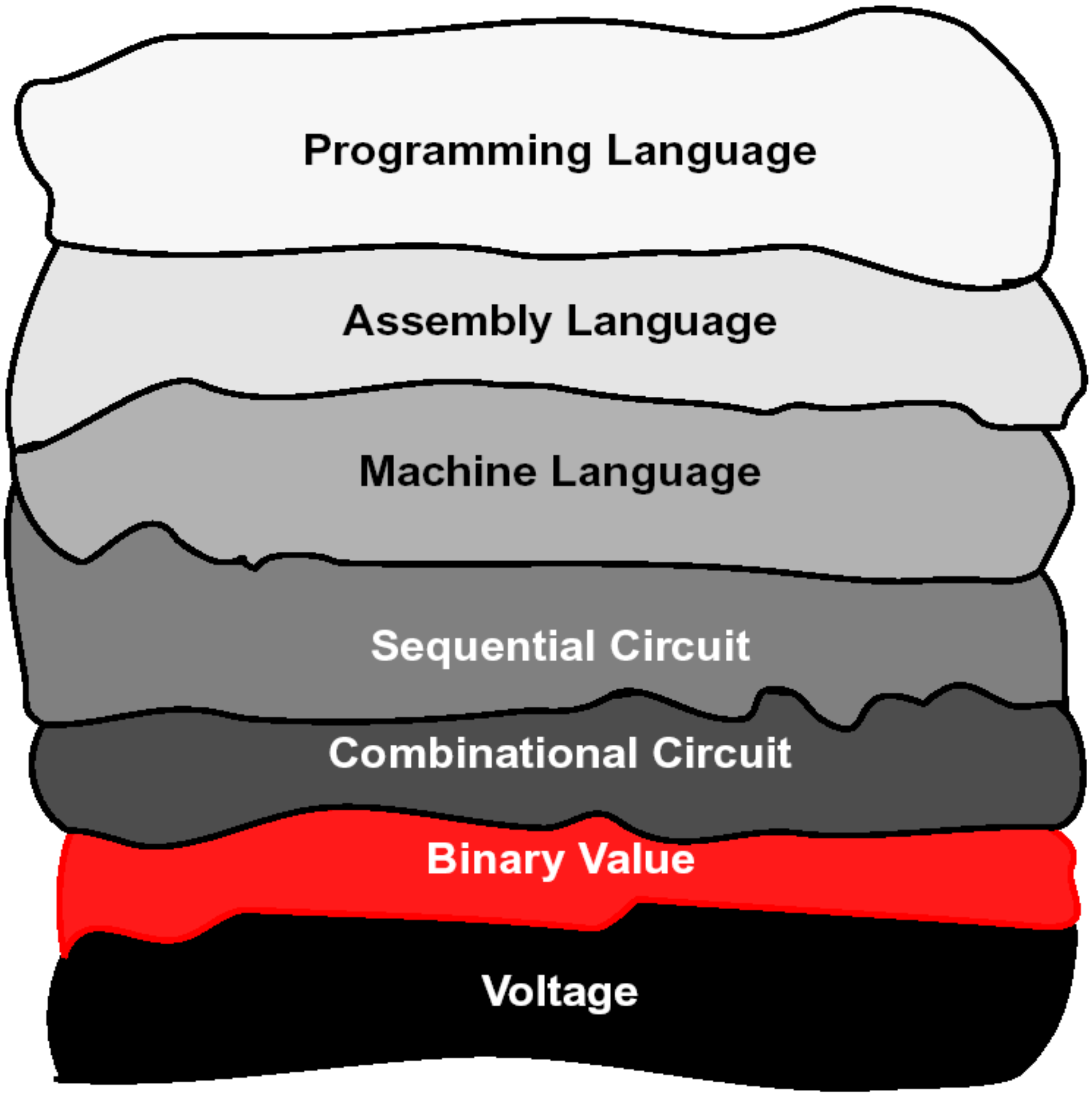
Representation

- Bits, Bytes, Words
- Unsigned Numbers
- Binary, Octal, Hex numbers
- Unsigned addition/subtraction

The Big Picture



Abstraction Hierarchy



Digital Abstraction

Binary digITS (bits) or binary numbers

- Smallest unit of information in a computer
- two values 0, 1
- abstraction of voltages

Bytes

- Sequences of 8-bits

Words

- Sequences of 32 bits (or 4 bytes) or 64 bits (or 8 bytes)
- architecture dependent

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Unsigned Representation

How to represent positive numbers?

- Simply use what we use every day
- Look at the number 89139

Positional Number Systems

- The same digits have different meaning depending on their position in the numeral.
- The value of a digit depends on the digit itself and of its position

Example

The decimal number $d_3d_2d_1d_0$ is

$$d_3 x10^3 + d_2 x10^2 + d_1 x10^1 + d_0 x10^0$$

More Generally

A number $d_{p-1}d_{p-2}...d_1d_0$ in *base* or *radix* b has the value

$$\sum_{i=0}^{p-1} b^i d_i$$

- except for leading zeros, the representation is unique
- leftmost digit: *most significant digit*
- rightmost digit: *least significant digit*

1001010101010000111010 1

Question

- What is the maximum value of d_i?

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1001010101010000111010 1

Question

- What is the maximum value of d_i?

Answer

- b-1

Binary Numbers

Numbers with radix 2 are called *binary numbers*

Only two digits: 0 and 1

Binary Number

110_2

Decimal Number

17_{10}

Binary Numbers

Numbers with radix 2 are called *binary numbers*

Only two digits: 0 and 1

Binary Number

110_2 6

Decimal Number

10001_2 17_{10}

Octal Numbers

Numbers with radix 8 are called octal numbers

- Digits: 0,1,2,3,4,5,6,7
- Each octal number can be written with exactly 3 bits

Binary Number	**Octal Number**
$111\ 110\ 010\ 001_2$	7621_8
$1\ 011\ 001\ 101\ 010_2$	13152_8

Hexadecimal Numbers

Numbers with radix 16 are called *hexadecimal (hex) numbers.*

- Digits: 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F
- Radix is a power of 2, so each hex digit can be written with exactly 4 bits
- Two hex digits describe the contents of a byte

Binary Number	**Hex Number**
$1111\ 0000\ 0000\ 1101_2$	$F00D_{16}$
$1000\ 0010\ 1110\ 1101_2$	$82ED_{16}$

How Many Toes Do You Have?

Binary	Decimal	Octal	Hex
0	0	0	0
1	1	1	1
10	2	2	2
11	3	3	3
100	4	4	4
101	5	5	5
110	6	6	6
111	7	7	7
1000	8	10	8
1001	9	11	9
1010	10	12	A
1011	11	13	B
1100	12	14	C
1101	13	15	D
1110	14	16	E
1111	15	17	F

General Conversions

With other bases, things are not that easy.

Two Steps:

- Convert to decimal
- Convert to the target base

Step 1: Convert to decimal by expansion

$1B07_{13} = 1x13^3 + 11x13^2 + 0x13^1 + 7x13^0$

$= 2197 + 1859 + 0 + 7$

$= 4063_{10}$

General Conversion

Now note that

$4063_{10} = ((4 \times 10 + 0) \times 10 + 6) \times 10 + 3$

So dividing by 10 gives us

- Remainder 3: the least significant digit
- Quotient: ((4x10 + 0) x 10 + 6)

General Conversion

Now note that

$4063_{10} = ((4x10 + 0) *10 + 6) *10 + 3$

So dividing by 10 gives us

- Remainder 3: the least significant digit
- Quotient: ((4x10 + 0) * 10 + 6)

This is true for any base

More generally, (b is the basis, d is in 0..b-1)

$D = ((...((d_{p-1})b + d_{p-2})b + ...)+ d_1)b + d_0$

So the quotient, when dividing by b, is

$Q = (...((d_{p-1})b + d_{p-2})b + ...)b + d_1$

and the remainder is d_0.

Successive divisions by the new base will yield the correct result.

More General Conversions

4063 ÷ 2 = 2031 remainder 1

2031 ÷ 2 = 1015 remainder 1

1015 ÷ 2 = 507 remainder 1

507 ÷ 2 = 253 remainder 1

253 ÷ 2 = 126 remainder 1

126 ÷ 2 = 63 remainder 0

63 ÷ 2 = 31 remainder 1

31 ÷ 2 = 15 remainder 1

15 ÷ 2 = 7 remainder 1

7 ÷ 2 = 3 remainder 1

3 ÷ 2 = 1 remainder 1

$$4063_{10} = 1111\ 1101\ 1111_2$$

Addition

$$
\begin{array}{r}
859_{10} \\
+ \quad 1257_{10} \\
\hline
\end{array}
$$

Addition

$$
\begin{array}{r}
859_{10} \\
+\ 1257_{10} \\
\hline
2116_{10}
\end{array}
$$

$$
\begin{array}{r}
157_{10} \\
+\ 306_{10} \\
\hline
463_{10}
\end{array}
$$

$$
\begin{array}{r}
56071_{8} \\
+\ 273645_{8} \\
\hline
351736_{8}
\end{array}
$$

Addition

$$\begin{array}{r} 859_{10} \\ +\ 1257_{10} \\ \hline 2116_{10} \end{array}$$

$$\begin{array}{r} 1101011011_{2} \\ +\ 10011101001_{2} \\ \hline \end{array}$$

$$\begin{array}{r} 157_{10} \\ +\ 306_{10} \\ \hline 463_{10} \end{array}$$

$$\begin{array}{r} 56071_{8} \\ +\ 273645_{8} \\ \hline 351736_{8} \end{array}$$

Addition

$$859_{10} + 1257_{10} = 2116_{10}$$

$$1101011011_2 + 10011101001_2 = 100001000100_2$$

$$157_{10} + 306_{10} = 463_{10}$$

$$10011101_2 + 100110010_2$$

$$56071_8 + 273645_8 = 351736_8$$

Addition

$$\begin{array}{r} 859_{10} \\ +\ 1257_{10} \\ \hline 2116_{10} \end{array}$$

$$\begin{array}{r} 1101011011_2 \\ +\ 10011101001_2 \\ \hline 100001000100_2 \end{array}$$

$$\begin{array}{r} 157_{10} \\ +\ 306_{10} \\ \hline 463_{10} \end{array}$$

$$\begin{array}{r} 10011101_2 \\ +\ 100110010_2 \\ \hline 111001111_2 \end{array}$$

$$\begin{array}{r} 4AF1_{16} \\ +\ C2B5_{16} \\ \hline \end{array}$$

$$\begin{array}{r} 56071_8 \\ +\ 273645_8 \\ \hline 351736_8 \end{array}$$

Addition

$$
\begin{array}{r} 859_{10} \\ +\ 1257_{10} \\ \hline 2116_{10} \end{array}
$$

$$
\begin{array}{r} 1101011011_{2} \\ +\ 10011101001_{2} \\ \hline 100001000100_{2} \end{array}
$$

$$
\begin{array}{r} 157_{10} \\ +\ 306_{10} \\ \hline 463_{10} \end{array}
$$

$$
\begin{array}{r} 10011101_{2} \\ +\ 100110010_{2} \\ \hline 111001111_{2} \end{array}
$$

$$
\begin{array}{r} 4AF1_{16} \\ +\ C2B5_{16} \\ \hline 11DA6_{16} \end{array}
$$

$$
\begin{array}{r} 56071_{8} \\ +\ 273645_{8} \\ \hline 351736_{8} \end{array}
$$

Subtraction

$$
\begin{array}{lr}
 & 229_{10} \\
- & \underline{\quad 46_{10}}
\end{array}
$$

Subtraction

$$\begin{array}{lr} & 229_{10} \\ - & 46_{10} \\ \hline & 183_{10} \end{array}$$

$$\begin{array}{lr} & 781_{10} \\ - & 538_{10} \\ \hline & 243_{10} \end{array}$$

$$\begin{array}{lr} & 73216_{8} \\ - & 20743_{8} \\ \hline & 52253_{8} \end{array}$$

$$\begin{array}{lr} & 11100101_{2} \\ - & 00101110_{2} \\ \hline \end{array}$$

Subtraction

$$\begin{array}{r} 229_{10} \\ -\ \ 46_{10} \\ \hline 183_{10} \end{array}$$

$$\begin{array}{r} 781_{10} \\ -\ 538_{10} \\ \hline 243_{10} \end{array}$$

$$\begin{array}{r} 73216_{8} \\ -\ 20743_{8} \\ \hline 52253_{8} \end{array}$$

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Subtraction

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$$\begin{array}{r} 1100001101_{2} \\ -\ 1000011010_{2} \\ \hline 0011110011_{2} \end{array}$$

$$\begin{array}{r} 19D9C_{16} \\ -\ DEAD_{16} \\ \hline \end{array}$$

Subtraction

$$\begin{array}{r} 229_{10} \\ -\ 46_{10} \\ \hline 183_{10} \end{array}$$

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$$\begin{array}{r} 19D9C_{16} \\ -\ DEAD_{16} \\ \hline BEEF_{16} \end{array}$$