Homework & Algorithm Assignment: Filter

CS123: Introduction to Computer Graphics

Due: Thursday, October 22 5:00 pm

This assignment is worth 7% of your final grade. Remember that collaboration is only allowed in accordance with our collaboration policy. You may talk about problems with others, but you must not take away written notes or answers from any collaboration sessions.

Tip: Be sure to run the signal processing applets before diving into this assignment:

`http://www.cs.brown.edu/exploratories/freeSoftware/catalogs/signalprocessing.html`

Your name: _____

CS login: _____

1 Theory (Homework)

Pixels

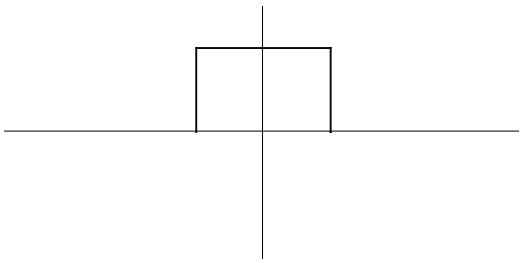
(5 points) Multiple choice: is a pixel a little square or circle?

a) No

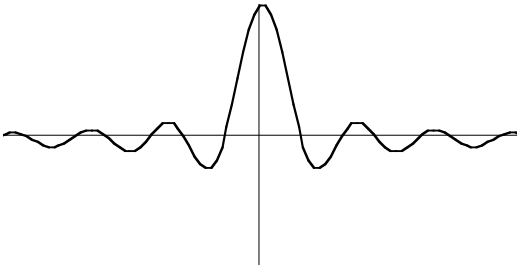
b) No

Duality of Domains

Recall that the dual of the box function in the spatial domain (Fig. 1) is the sinc function in the frequency domain (Figure 2).

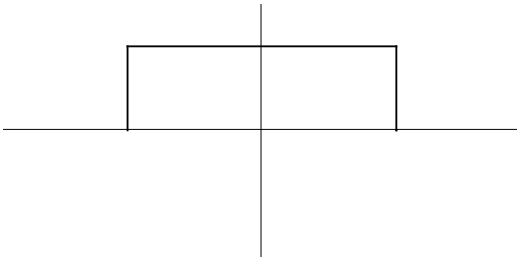


(Fig. 1)

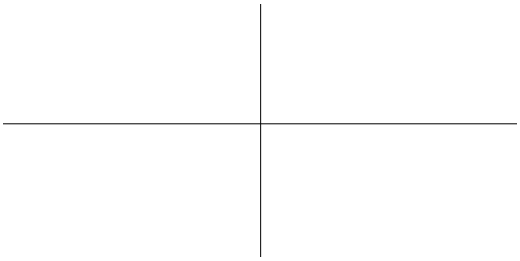


(Fig. 2)

(10 points) Sketch the dual of function (a), below, in the space provided. Your sketch doesn't need to be accurate, but it should be different from Figure 2, above. If you don't think your sketch is clear, please write a brief explanation in the margin.



(a)



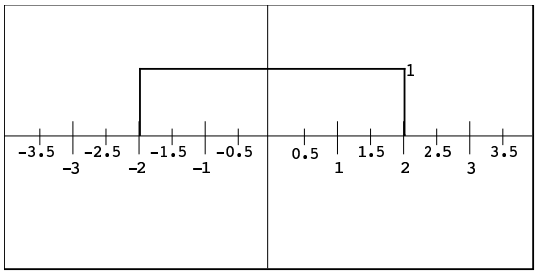
(b)

(10 points) What do we use to approximate the sinc function, and why do we make this approximation when translating these theoretical concepts to code?

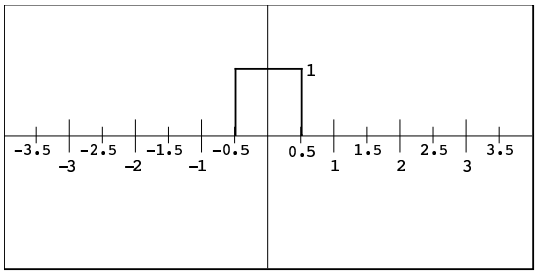
Convolution

Consider the following function and filter.

Function



Filter



(3 points each) What is the value of the function convolved with the filter at each of the following values of x?

$x = 3$ _____

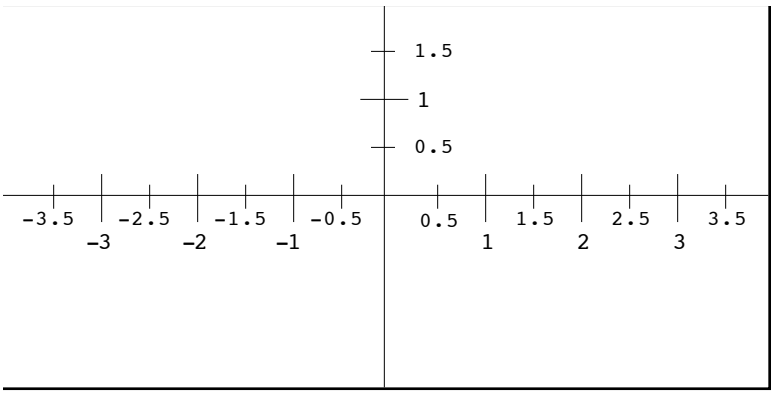
$x = 2.5$ _____

$x = 2.25$ _____

$x = 2$ _____

$x = 1.5$ _____

(10 points) Draw the function convolved with the filter in the blank graph provided below:



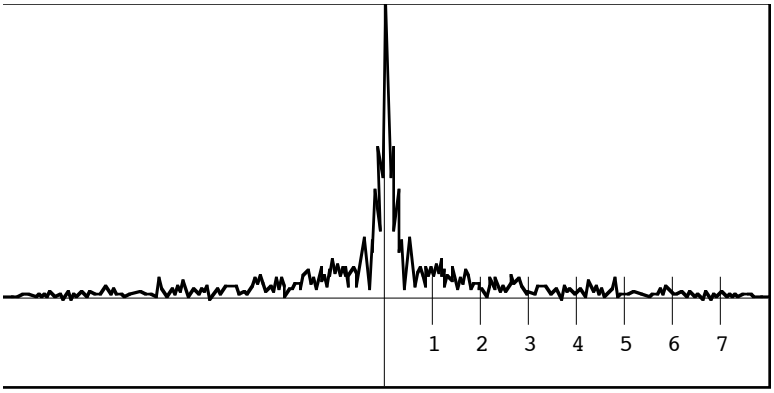
(10 points) Let $F(x)$ and $G(x)$ be the frequency domain duals of spatial domain functions $f(x)$ and $g(x)$, respectively. Fill in the right-hand side of the following equation with an equivalent operation involving $F(x)$ and $G(x)$. Note that $*$ is the convolution operator.

$$f(x) * g(x) =$$

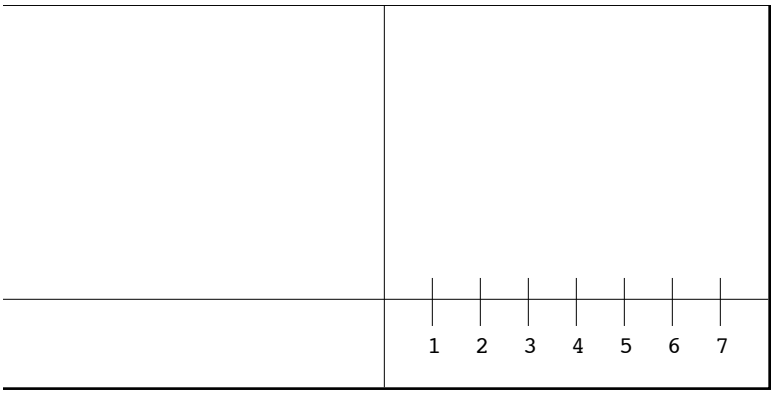
Prefiltering

(10 points) If we are sampling at a frequency of n samples per unit, what is the largest frequency we can represent, according to the Nyquist limit?

Examine this frequency domain plot of the infamous Mandrill:



(10 points) If you were going to sample this signal at a rate of 4 samples per unit, to avoid aliasing you would use what we know about the Nyquist limit to prefilter it. Sketch the new frequency plot after this filtering step:



Anti-Aliasing

(10 points) What is the effect of an ideal blur (anti-aliasing) filter in the frequency domain?

(10 points) What is the ideal blur filter in the spatial domain?