

# CS138 Homework Assignment 4

*Due April 23, 2009*

1. Slide XX-8 showed a solution for a self-stabilizing token ring, in which each of the  $n$  nodes has  $k \geq n$  states; assume  $n > 1$ . Since Dijkstra didn't bother proving it correct, it's up to you. We'll define the system to be stable when it's the case that only one node's guard is currently true, and whenever the system changes global state legally (i.e., without being zapped), it goes to a global state in which the next node's guard becomes the only guard whose value is true. Assume that the nodes are numbered such that the distinguished node is 0 and the others are numbered successively from 1 to  $n-1$ .
  - a. Explain why it is that at any particular moment, at least one guard must be true, even if the system has been zapped.
  - b. Show that if all nodes have the same value for their states, the system is stable.
  - c. Show that if node 0's state is greater than those of all other nodes, the system will necessarily reach a stable global state.
  - d. Assume now that each node's state value is an unbounded integer (i.e.,  $k$  is infinite). Show that, regardless of its current state, the system will necessarily reach a global state in which node 0's state is greater than those of all others.
  - e. Redo part d, this time assuming  $k \geq n$ .
  - f. Show that the system won't necessarily ever enter a stable state after being zapped if  $k < n$ .