

Homework 2

Heuristic Search

Due: 7:00pm on 2/20/08

Problem 2.1

Before you begin, read the slide handout (or at least the parts that explain the game, A*, and Manhattan distance). Now perform A* by hand on the following sliding tiles problem. If there is a tie as to which node on the “fringe” to expand next (they have the same f value), pick the node you get to by moving the lower numbered tile. You may use the demo to check your work on this problem, but if you only use the demo you will be missing a large part of the search tree.

1	2	3	4
5	7	15	8
9	6	11	0
13	10	14	12

Your handin should consist of a fairly large tree where each node has

- A sketch of the current state of the board, similar to the one above
- The current G value
- The current H value

This can either be done by hand or with a program like Dia, but be sure the end result is readable and clear

You only need to go to a depth of 4 (the initial state is depth 0).

Problem 2.2

Which of the following are admissible, given admissible heuristics h_1, h_2 ? Which of the following are consistent, given consistent heuristics h_1, h_2 ?

- $h(n) = \min\{h_1(n), h_2(n)\}$
- $h(n) = wh_1(n) + (1 - w)h_2(n)$, where $0 \leq w \leq 1$
- $h(n) = \max\{h_1(n), h_2(n)\}$

Please provide proofs.

Problem 2.3

Consider the algorithm wA^* , which is a variant of A^* search that uses the following weighted cost function: for some $w \geq 1$,

$$f_w(n) = g(n) + wh(n)$$

As usual, g is the cost from the root node to n and h is an admissible heuristic.

- Prove that the goal node m^* returned by wA^* search on trees is within a factor of w of the optimal goal n^* : i.e., $g(m^*) \leq wg(n^*)$.
- The wA^* algorithm uses a weighted cost function that increases the value of the heuristic function h , hoping to proceed more directly towards a goal. An alternative is to ignore g entirely, simply letting $f = h$. Give two advantages of wA^* over this alternative.

Problem 2.4

Connect Four is a classic struggle of red (X) against black (O) for domination in a gridded world. It is also a fun game (if you are unfamiliar with the rules, ask google). You are red (X). What is your next move? Argue why. Describe a system for determining your next move.

				O		
X				X		
O	O		O	X	O	
O	X		X	O	O	X

Problem 2.5

Given a finite set S and a binary operation $* : S \times S \rightarrow S$, the tuple $(S, *)$ is called a group if and only if the following properties hold:

- Neutral Element: $\exists n \in S \forall a \in S : n * a = a = a * n$
- Associative: $\forall a, b, c \in S : (a * b) * c = a * (b * c)$
- Inverse: $\forall a \in S : \exists b \in S : a * b = n = b * a$

Prove the following:

- The neutral element is unique.
- $\forall a \in S$, it's inverse element b with $b * a = n = a * b$ is unique.

Each of these can be done in less than 5 lines.