CS141 **Introduction to AI** **Professor Meinolf Sellmann**

Homework 4
Propositional Logic

Due: 6:00pm on 10/14/09

Problem 4.1

Yuri and Ilke, the darling CS141 TA duo, have been exploring the Wumpus world together (described on page 197 of Russell and Norvig). Being a novice explorers, the grand Wumpus Master chose a world in which no two pits are directly adjacent. Despite this restriction, our cold, tired and frightened TAs need your help to safely navigate the Wumpus world!

Yuri and Ilke have made the following map to help you in this task:

BREEZE	???	???	???
STENCH	???	???	???
OK	STENCH	BREEZE	???
OK	BREEZE	???	???

a. Where is the wumpus?

b. Where are the definite pits? Prove this. Be sure to formulate your proof in propositional logic.

Solution:

a. The Wumpus cavorts about vaunting its stench in square (2,3). We know, From $S_{1,3}$ and $S_{2,2}$ that it can only be in (1,2) or (2,3). Because we've already visited (1,2) and know it to be safe, the only remaining possibility, wumpus-wise, is (2,3).

b. There are known pit in (2,4) and (3,1)

$$\frac{B_{1,4} \leftarrow P_{2,4} \bigvee P_{1,3}, B_{1,4}}{P_{2,4} \bigvee P_{1,3}}$$

$$\frac{P2,4 \bigvee P_{1,3}, \neg P_{1,3}}{P_{2,4}}$$

And (3,1) has a pit as well:

$$\frac{B_{2,1} \leftarrow P_{1,1} \bigvee P_{2,2} \bigvee P_{3,1}, B_{2,1}}{P_{1,1} \bigvee P_{2,2} \bigvee P_{3,1}}$$

$$\frac{P_{1,1} \bigvee P_{2,2} \bigvee P_{3,1}, \neg P_{1,1}}{P_{2,2} \bigvee P_{3,1}}$$

$$\frac{P_{2,2} \bigvee P_{3,1}, \neg P_{2,2}}{P_{3,1}}$$

Problem 4.2

For what assignemtns of TRUE or FALSE to P, Q, R, and S is the following sentence true?

$$\neg P \vee Q \wedge R \Rightarrow \texttt{ TRUE } \Leftrightarrow \texttt{S}$$

Solution:

$$X \Rightarrow Y \Leftrightarrow \neg X \vee Y \text{ so } X \Rightarrow \texttt{ TRUE } \Leftrightarrow \texttt{ TRUE}$$

Then everything left of the $\Leftrightarrow$ is always TRUE
So we have TRUE $\Leftrightarrow$ S which is equivalent to S

The clause is true if and only if S is TRUE

Problem 4.3

Show that it is possible to define the meaning of $\vee$, $\rightarrow$, $\leftarrow$, and $\leftrightarrow$ in terms of $\neg$ and $\wedge$. In other words, show that the subset $\{\neg, \wedge\}$ suffices to express all the formulas of propositional logic. Are there other subsets of the set of connectives $\{\neg, \vee, \wedge, \rightarrow, \leftarrow, \leftrightarrow\}$ that exhibit this property?

Solution:

$A \vee B = (\neg A \wedge \neg B)$

$A \leftarrow B = \neg(A \wedge \neg B)$

$A \rightarrow B = \neg(\neg A \wedge B)$

$A \leftrightarrow B = \neg(A \wedge \neg B) \wedge \neg(\neg A \wedge B)$

We expected truth tables to back up each of these statements.

There are many subsets of the set of logical connectives that are sufficient to express all the formulas of propositional logic; in particular, $\{\neg, \wedge\}$, $\{\neg, \vee\}$, $\{\neg, \leftarrow\}$, and $\{\neg, \rightarrow\}$, as well as any superset of these sets. Note, however, that $\{\neg, \leftrightarrow\}$ is not such a subset.

Problem 4.4

Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

Solution:

Convert each implication in the paragraph to a disjunction of literals. Your knowledge base is the conjunction of all these disjunctions (and is therefore in conjunctive normal form). Now apply the resolution algorithm to this knowledge base and the sentences Mythical, Magical and Horned individually.

Knowledge Base:

- $\neg Mythical \vee Immortal$
- $Mythical \vee \neg Immortal$
- $Mythical \vee Mammal$
- $\neg Immortal \vee Horned$
- $\neg Mammal \vee Horned$
- $\neg Horned \vee Magical$

Mythical:

a. Assume $\neg Mythical$

b. $Mythical \vee \neg Immortal, \neg Mythical \rightarrow \neg Immortal$

c. $Mythical \vee Mammal, \neg Mythical \rightarrow Mammal$

d. $\neg Mammal \vee Horned, Mammal \rightarrow Horned$

e. $\neg Horned \vee Magical, Horned \rightarrow Magical$

f. At this point, all clauses in our KB are now satisfied. So we're done, and we can't prove Mythical.

Satisfying assignment:

Mythical	Magical	Immortal	Mammal	Horned
FALSE	TRUE	FALSE	TRUE	TRUE

Magical:

a. Assume $\neg Magical$

b. $\neg Horned \vee Magical, \neg Magical \rightarrow \neg Horned$

c. $\neg Immortal \vee Horned, \neg Horned \rightarrow \neg Immortal$

d. $\neg Mythical \vee Immortal, Immortal \rightarrow \neg Mythical$

e. $\neg Mammal \vee Horned, \neg Horned \rightarrow \neg Mammal$

f. $Mythical \vee Mammal, \neg Mammal \rightarrow Mythical$

g. $Mythical, \neg Mythical \rightarrow \{\}$

Assuming $\neg Magical$ leads to a contradiction. So $Magical$ is true.

Horned: This proof is the same as for Magical, except that instead of deriving $\neg Horned$ in step b, you can simply assume it. The same contradiction is then reached, so $Horned$ must be true.

Problem 4.5

You are given the following SAT problem instance:

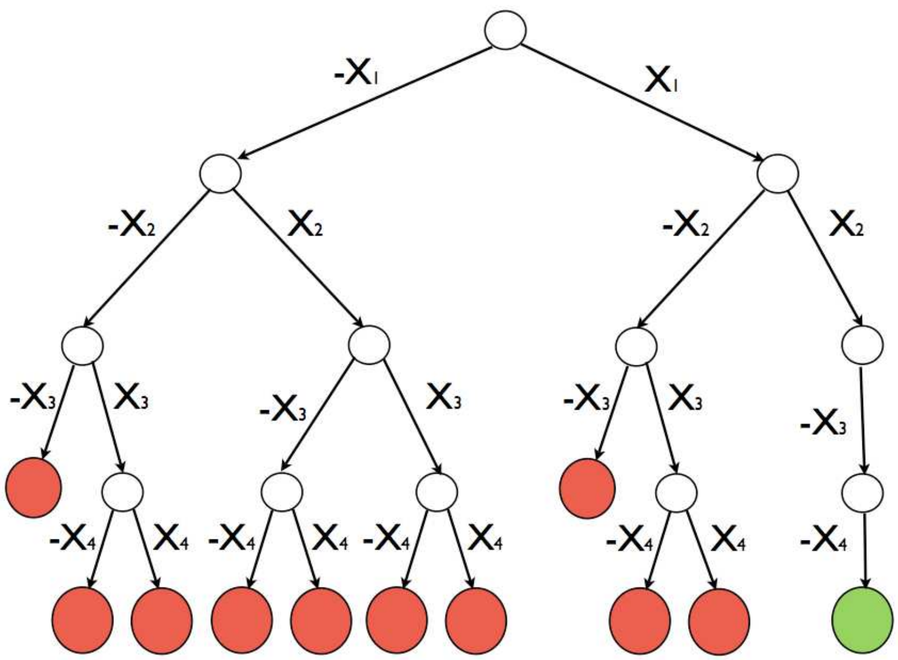
$$(x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_3 \vee x_4) \wedge (x_1 \vee x_2 \vee \neg x_4) \wedge (x_1 \vee x_3 \vee x_4) \wedge$$
$$(\neg x_2 \vee x_3 \vee \neg x_4) \wedge (x_1 \vee \neg x_3 \vee \neg x_4) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (x_2 \vee \neg x_3 \vee x_4) \wedge$$
$$(\neg x_1 \vee \neg x_3 \vee \neg x_4) \wedge (x_1 \vee \neg x_2 \vee x_4)$$

(a) Your branching heuristic tells the solver to branch on the variable with the smallest index first, and setting that variable to false. Draw the search tree of a depth first algorithm for the given SAT problem and the given heuristic.

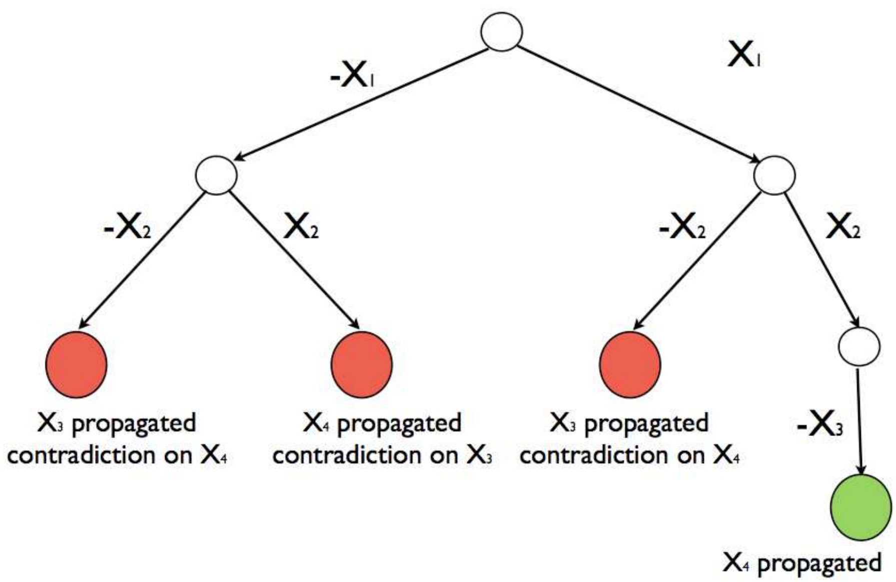
(b) You may have noticed that during the search, some clauses ended up having only one variable remaining (referred to as a *unary* clause). This means that in order to make the corresponding clauses evaluate to TRUE, that variable had to be assigned a specific value. Furthermore, with the assignment made, you might find that other clauses become unary as well and you can automatically assign some more variables. This process is called *inference*. Draw the search tree for the given SAT problem and the given heuristic, but this time applying inference at each decision node.

(c) After reaching a leaf node, you have to backtrack and try an alternate subtree. Explain what information can be learned after reaching a leaf node that can be used to speed up your remaining search. Using your described approach, draw the new search tree for the given SAT problem and the given heuristic.

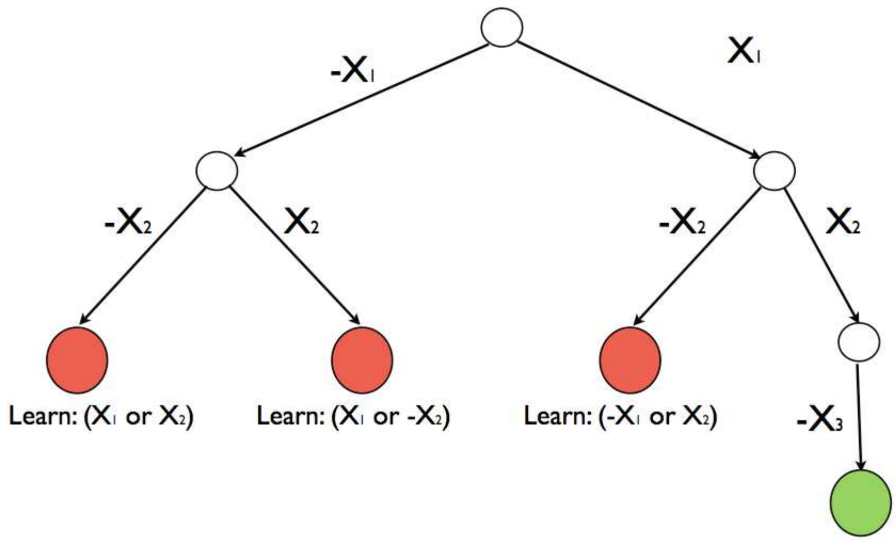
Solution:



(a)



(b)



(c)