CS141 **Introduction to AI** **Professor Meinolf Sellmann**

Homework 4
Propositional Logic

Due: 6:00pm on 10/14/09

Problem 4.1

Yuri and Ilke, the darling CS141 TA duo, have been exploring the Wumpus world together (described on page 197 of Russell and Norvig). Being a novice explorers, the grand Wumpus Master chose a world in which no two pits are directly adjacent. Despite this restriction, our cold, tired and frightened TAs need your help to safely navigate the Wumpus world!

Yuri and Ilke have made the following map to help you in this task:

BREEZE	???	???	???
STENCH	???	???	???
OK	STENCH	BREEZE	???
OK	BREEZE	???	???

a. Where is the wumpus?

b. Where are the definite pits? Prove this. Be sure to formulate your proof in propositional logic.

Problem 4.2

For what assignemtns of TRUE or FALSE to P, Q, R, and S is the following sentence true?

$$\neg P \vee Q \wedge R \Rightarrow \texttt{TRUE} \Leftrightarrow \texttt{S}$$

Problem 4.3

Show that it is possible to define the meaning of $\vee$, $\rightarrow$, $\leftarrow$, and $\leftrightarrow$ in terms of $\neg$ and $\wedge$. In other words, show that the subset $\{\neg, \wedge\}$ suffices to express all the formulas of propositional logic. Are there other subsets of the set of connectives $\{\neg, \vee, \wedge, \rightarrow, \leftarrow, \leftrightarrow\}$ that exhibit this property?

Problem 4.4

Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

Problem 4.5

You are given the following SAT problem instance:

$$(x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_3 \vee x_4) \wedge (x_1 \vee x_2 \vee \neg x_4) \wedge (x_1 \vee x_3 \vee x_4) \wedge$$
$$(\neg x_2 \vee x_3 \vee \neg x_4) \wedge (x_1 \vee \neg x_3 \vee \neg x_4) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (x_2 \vee \neg x_3 \vee x_4) \wedge$$
$$(\neg x_1 \vee \neg x_3 \vee \neg x_4) \wedge (x_1 \vee \neg x_2 \vee x_4)$$

(a) Your branching heuristic tells the solver to branch on the variable with the smallest index first, and setting that variable to false. Draw the search tree of a depth first algorithm for the given SAT problem and the given heuristic.

(b) You may have noticed that during the search, some clauses ended up having only one variable remaining (referred to as a *unary* clause). This means that in order to make the corresponding clauses evaluate to TRUE, that variable had to be assigned a specific value. Furthermore, with the assignment made, you might find that other clauses become unary as well and you can automatically assign some more variables. This process is called *inference*. Draw the search tree for the given SAT problem and the given heuristic, but this time applying inference at each decision node.

(c) After reaching a leaf node, you have to backtrack and try an alternate subtree. Explain what information can be learned after reaching a leaf node that can be used to speed up your remaining search. Using your described approach, draw the new search tree for the given SAT problem and the given heuristic.