Homework 5
Logic and Learning

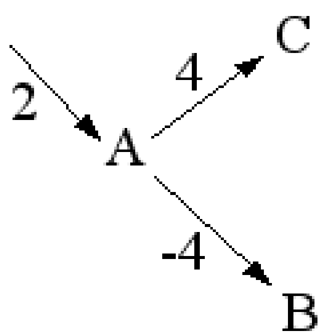
Due: 6:00pm on 10/28/09

Problem 5.1

Given the following SAT instance:

(-1 - 4 + 5) (-1 + 3 - 4) (+4 - 1 + 6) (+5 - 2) (+2 - 3 + 4) (-1 + 2) (-1 + 5 - 6) (+1 - 6) (+6 - 4 -5) (-1 - 6 + 4) (-3 - 2 - 4) (+4 + 1)

and the following branching graph:



For each node (A, B, and C) generate all clauses that you can learn and the associated inference graph. Use the results from node A when working on nodes B and C.

Problem 5.2

Perform unit propagation on the following sentence. Show each step and determine all models that makes the sentence true.

$$(\neg P \vee Q \vee S), (\neg P \vee \neg R \vee S), (P), (\neg Q \vee R), (\neg S)$$

Problem 5.3

Given a finite set S and a binary operation $* : S \times S \rightarrow S$, the tuple (S,*) is called a group if and only if the following properties hold:

- Neutral Element: $\exists n \in S\ \forall a \in S : n * a = a = a * n$
- Associative: $\forall a, b, c \in S : (a * b) * c = a * (b * c)$

- Inverse: $\forall a \in S : \exists b \in S : a * b = n = b * a$

a. Given the set $\{0, 1\}$ and an operation $* : \{0, 1\} \times \{0, 1\} \rightarrow \{0, 1\}$, express the fact that $(\{0, 1\}, *)$ is a group in propositional logic.

b. Given a refutation complete algorithm for PL, how could you use the solver to show that the neutral element of any group over $\{0, 1\}$ is unique?

c. How could you show that the inverse element is unique?

Problem 5.4

Define a consistent vocabulary and represent the following sentences in first-order logic:[1]

a. Every person who buys a policy is smart.

b. No person buys an expensive policy.

c. There is an agent who sells policies only to people who are not insured.

d. There is a barber who shaves all men in town who do not shave themselves.

e. A person born in the UK, each of whose parents is a UK citizen or a UK resident, is a UK citizen by birth.

f. A person born outside the UK, one of whose parents is a UK citizen by birth, is a UK citizen by descent.

g. Politicians can fool some people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time.

[1]From Russell and Norvig 8.6, e-k.