

Homework 6

A Logic Sandwich

Due: 6:00pm on 11/04/09

Problem 6.1

Read Chapter 14 from Russel and Norvig.

Problem 6.2

You can represent a queue as an abstract data structure in predicate logic. The following axioms define the basic operations enqueue, dequeue, and first on a queue:

- a. $Queue(\epsilon)$
- b. $\forall x : Queue(x) \Rightarrow x = \epsilon \vee \exists y, z : Queue(y) \wedge x = Enqueue(y, z)$
- c. $\forall x, y : Queue(x) \Rightarrow Queue(Enqueue(x, y))$
- d. $\forall y : Front(Enqueue(\epsilon, y)) = y$
- e. $\forall x, y : Queue(x) \Rightarrow x = \epsilon \vee Front(x) = Front(Enqueue(x, y))$
- f. $\forall y : Dequeue(Enqueue(\epsilon, y)) = \epsilon$
- g. $\forall x, y : Queue(x) \Rightarrow x = \epsilon \vee Dequeue(Enqueue(x, y)) = Enqueue(Dequeue(x), y)$

Using the above as the basis for your queue:

- a. Give the axioms for the function, Append(x,y) that takes the contents of one queue and appends it onto queue in predicate logic.
- b. Give the axioms for Reverse(x), which reverses the order of the elements in a queue, in predicate logic using Append from part a.

Problem 6.3

- a. Transform the following into Skolem normal form:

$$\alpha = \forall x \forall y \forall z [f(f(x, y), z) = f(x, f(y, z)) \wedge \exists e \forall x (f(x, e) = x \wedge \exists x' (f(x, x') = e))]$$

- b. Unify the following atomic sentences or give an argument why this is not possible. P is a predicate, a and b are constants, f, g and h are functions and x, y, z, x_0, x_1, x_2 are variables. Assume that standardizing apart has already been performed, i.e., unify the pair of sentences without performing any variable renaming.

(a) $\{p(f(f(f(a, x_0), x_1), x_2)), p(f(x_2, f(x_1, f(x_0, a))))\}$

(b) $\{p(x), p(f(x))\}$

(c) $\{p(x, h(x, y), y), p(x, z, b)\}$

(d) $\{p(x, x, y, y), p(x, f(y), y, f(x))\}$

Problem 6.4

Matt is attempting to cook mushrooms for the TAs to enjoy during a grading meeting. This might have some unexpected consequences.

Here is what we know in predicate logic:

Let Matt, Yuri and Ilke be constants. Let x and y be variables. Let Meet, MustEatMushroom, HatesMatt, and LikesEachOther be functions.

$$\begin{aligned} & \text{Meet}(\text{Yuri}, \text{Matt}) \wedge \\ & \text{Meet}(\text{Ilke}, \text{Matt}) \wedge \\ & (\forall x, y (\text{Meet}(x, \text{Matt}) \Rightarrow \text{MustEatMushroom}(x))) \wedge \\ & (\text{MustEatMushroom}(x) \Rightarrow \text{HatesMatt}(x)) \wedge \\ & (\text{HatesMatt}(x) \wedge \text{HatesMatt}(y) \Rightarrow \text{LikesEachOther}(x, y)) \end{aligned}$$

- a. Bring the above to Prenex normal form
- b. Show $\text{LikesEachOther}(\text{Yuri}, \text{Ilke})$