Homework 2: Local Search and CSPs

Due: 5:00 PM, Feb 23, 2009

Contents

Objectives

By the end of this homework, you will know:

1. that Lewis Carroll was a fan of CSPs
2. what a minimum spanning tree is
3. what the classic satisfiability problem is

Practice

1 Lewis Carroll

The following puzzle is due to Lewis Carroll. A small street has five houses, each one a different color, and each one housing a man of a different nationality. Each man has a different profession, likes a different drink, and has a different pet animal. The following information is given:

```
The Englishman lives in the red house.
The Spaniard has a dog.
The Japanese man is a painter.
The Italian drinks tea.
The Norwegian lives in the first house on the left.
The owner of the green house drinks coffee.
The green house is on the right side of the white house.
The sculptor breeds snails.
The diplomat lives in the yellow house.
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They drink milk in the middle house.
The Norwegian lives next door to the blue house.
The violinist drinks fruit juice.
The fox is in the house next to the doctor's.
The horse is in the house next to the diplomat's.
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Who has the zebra, and who drinks water?

(a) Formulate this puzzle as a CSP.

(b) Solve this puzzle.

Problems

2 A Variant of Simulated Annealing

Consider the following best-improvement variant of simulated annealing: at each step, starting at state x, evaluate the objective function at all neighbors $y_1, \ldots, y_n$. If any neighbor improves the value of the objective function, then move greedily to the best neighbor: set x equal to an $\arg\min_{y \in \{y_1, \ldots, y_n\}} \text{Obj}(y)$. Otherwise, if no neighbor improves the value of the objective function, then stochastically move to some neighbor y with probability $p \sim e^{-\Delta(x,y)/T}$, for $T > 0$, where $\Delta(x, y) = \text{Obj}(y) - \text{Obj}(x)$. (The probabilities are normalized to sum to 1, so that the algorithm is guaranteed to accept some move.)

Task: Describe a search space in which this algorithm outperforms classic simulated annealing. Use no more than five states.

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Task: Describe the behavior of this algorithm as $T \to \infty$ in terms of heuristics and/or related optimization algorithms.

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3 Minimum Spanning Trees

Given a graph $G = (V, E, c)$, where V is a set of vertices, E is a set of edges, and $c : E \to \mathbb{R}^+$ associates costs with edges, a *spanning tree* is a tree formed from edges in the graph that contains all vertices. A *minimum spanning tree* (MST) is a spanning tree for which the sum of edge costs is minimal.

Kruskal proposed a greedy algorithm for constructing minimum spanning trees: (i) sort edges from least to greatest cost; and (ii) greedily add edges in order, but if doing so creates a cycle, then discard the edge in question. This greedy algorithm is in fact optimal: it outputs a spanning tree of minimal cost.

Task: Argue that MST is an admissible heuristic in TSP: *i.e.*, the cost of a MST is a lower bound on the cost of an optimal solution in TSP.

Task: Argue that twice the cost of a MST is an upper bound on the cost of an optimal solution in TSP, assuming the cost function satisfies the triangle inequality: $c(x,y) + c(y,z) \geq c(x,z)$, for all edges $(x,y),(y,z),(x,z) \in E$.

4 Cryptoarithmetic

Task: Solve the following cryptoarithmetic puzzle.

```
  S E N D
+ M O R E
---------
M O N E Y
```

5 N-Queens

Task: Formulate the n-queens problem as a CSP.

Task: Show that n-queens is an instance of SAT (so that n-queens is NP-hard): i.e., formulate the n-queens problem in the language of propositional logic.