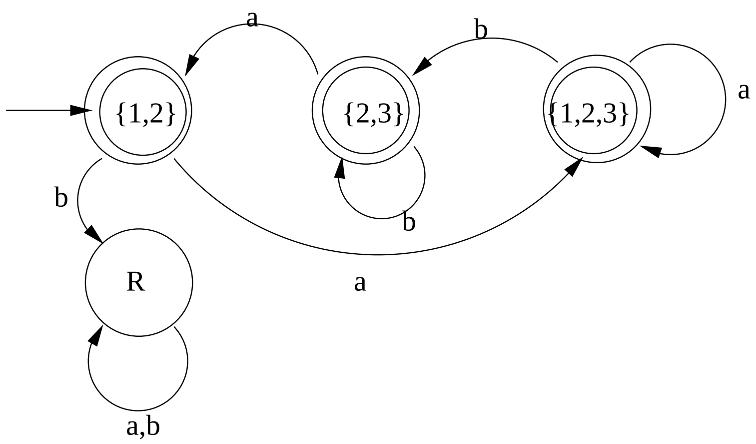
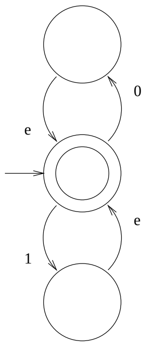
Homework 1: Solutions

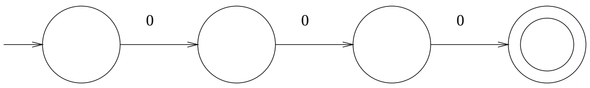
Problem 1.



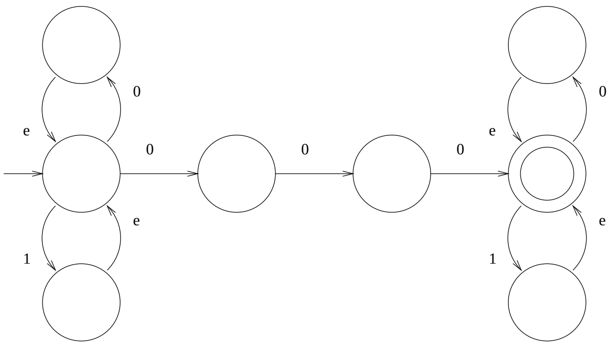
Problem 2a.



accepts (0 U 1)*

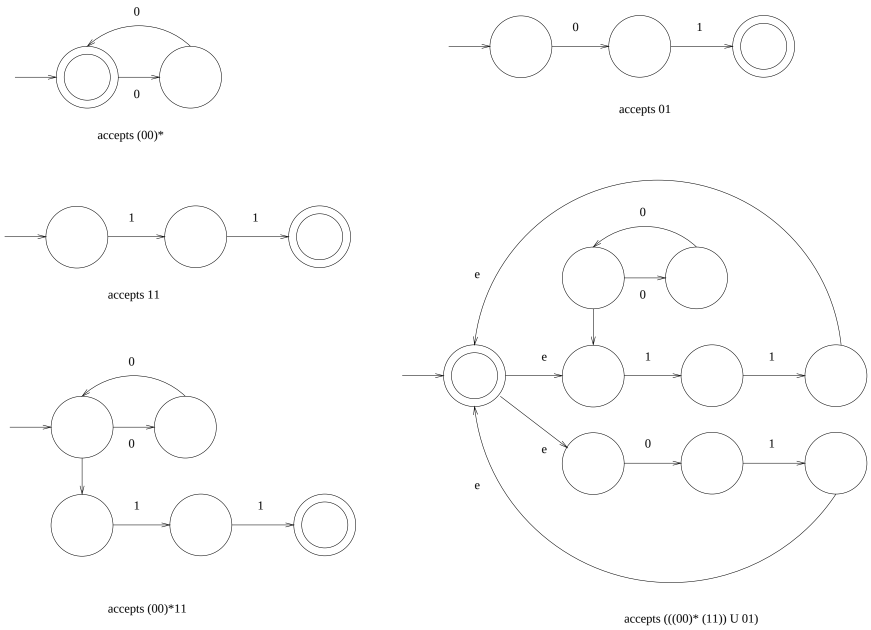


accepts 000

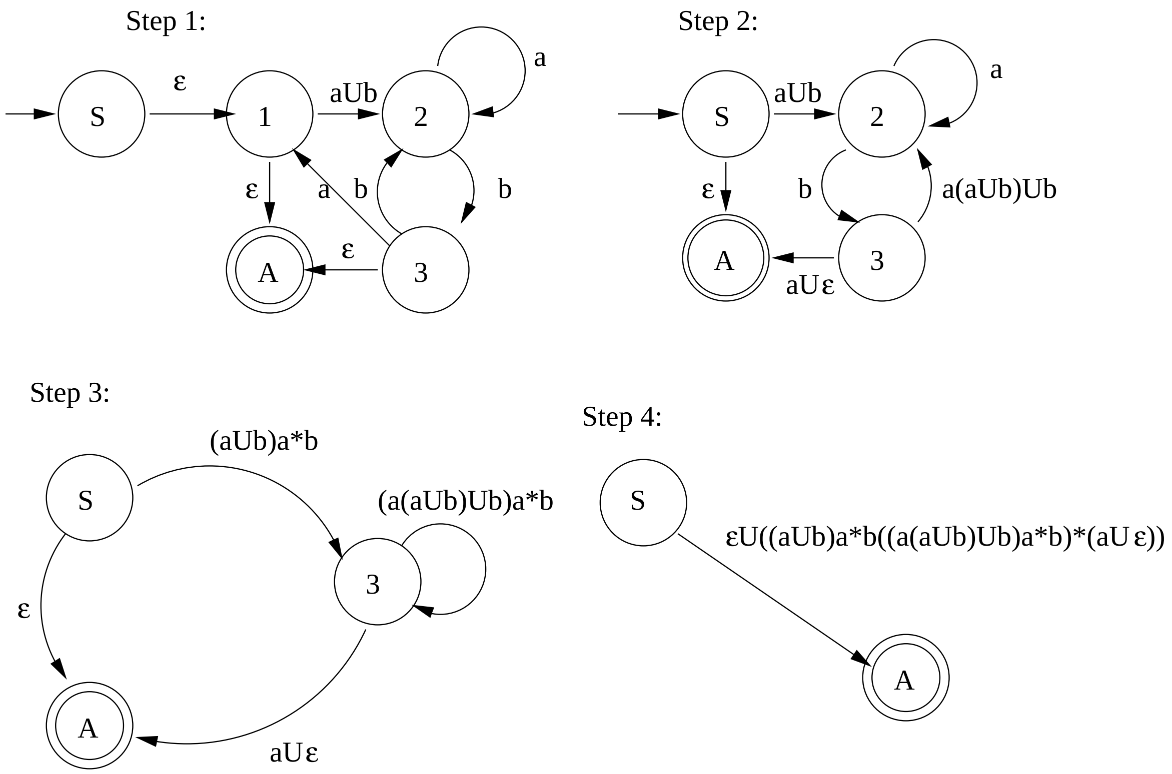


accepts (0 U 1)*000(0 U 1)*

Problem 2b.



Problem 3.



Problem 4.

Consider the DFA $D_n = (Q, \Sigma, \delta, q_0, F)$ where

$Q = \{q_i | 0 \leq i < n\}$

$\Sigma =$ a

$\delta = (q_i, \text{a}) \rightarrow q_{(i+1)\ mod\ n}$

$F = \{q_0\}$

D_n is structured as a ring, so that after reading j characters D_n is in state $\text{q}_{j\ mod\ n}$. D_n is in the accept state q_0 after reading cn characters, so D_n accepts the language $B_n = \{\text{a}^k | k \text{ is a multiple of } n\}$. B_n is regular because it is accepted by a DFA.

Problem 5.

Given a context-free langauge A and a regular language B, there is a PDA $P = (Q_1, \Sigma_1, \Gamma, \delta_1, q_1, F_1)$ that accepts A and DFA $D = (Q_2, \Sigma_2, \delta_2, q_2, F_2)$ that accepts B. Let PDA $N = (Q, \Sigma, \Gamma, \delta, q_0, F)$ where

$Q = Q_1 \times Q_2$

$\Sigma = \Sigma_1 \cap \Sigma_2$

$\delta = \{((q_a, q_b), s, t) \rightarrow S | (q_a, q_b) \in Q, s \in \Sigma_\epsilon, t \in \Gamma_\epsilon, (q_a, s) \rightarrow q_a\text{'} \in \delta_1, S = \{((q_a\text{'}, q_b\text{'}), t\text{'}) | (q_b\text{'}, t\text{'}) \in P, (q_b, s, t) \rightarrow P \in \delta_2\}\}$

$q_0 = (q_1, q_2)$

$F = \{(q_r, q_s) | q_r \in F_1, q_s \in F_2\}$

Each state in N corresponds to a state in P and a state in D, and each step of a computation of N simulates a step of a computation of P and of D in parallel. N ends in an accepting state if and only if P and D end in accepting states, so the language C accepted by N consists of all strings accepted by both P and D, that is, $C = A \cap B$. C is context-free because N is a PDA, so the intersection of a regular language and a context-free language is context-free.

Let language R be regular and language L be context-free. $L\text{-}R = L \cap \overline{R}$. $\overline{R}$ is regular because the regular languages are closed under complement. Therefore $L\text{-}R$ is the intersection of a regular language and a context-free language, so $L\text{-}R$ is context-free.