Homework 3

Due: Friday, Mar. 12, at 9:00pm in the handin bin located on 2^{nd} floor CIT

Collaboration is allowed only by terms of the policy. All problems are worth 20 points. Tom will hold TA hours Thursday 8-9PM, Dave Friday 3-4PM in the TA annex room on the second floor of the CIT.

Problem 1.

Let $INFINITE_{DFA} = \{\langle A\rangle | L(A) \text{ is an infinite language}\}$. Show that $INFINITE_{DFA}$ is decidable.

Problem 2.

Let $T = \{(i,j,k) | i,j,k \in \mathcal{N}\}$. Show that T is countable.

Problem 3.

Let A be a Turing-recognizable language consisting of descriptions of Turing machines, $\{\langle M_1\rangle, \langle M_2\rangle, ...\}$, where every M_i is a decider. Prove that some decidalbe language D is not decided by any decider M_i whose description appears in A.

HINT: You may find it helpful to consider an enumerator for A.

Problem 4.

A *useless state* in a Turing machine is one that is never entered on any input string. Consider the problem of testing whether a Turing machine has any useless states. Formulate this problem as a language and show that it is undecidable.

Problem 5.

Consider the problem of testing whether a Turing machine M on input w ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and show that it is undecidable.