Homework 3: Solutions

Problem 1.

Let $INFINITE_{DFA} = \{\langle A\rangle | L(A) \text{ is an infinite language}\}$. Show that $INFINITE_{DFA}$ is decidable.

Answer
We construct a Turing machine M that decides the language $INFINITE_{DFA}$ as follows. On input $\langle A\rangle$, machine M first figures out the number of states in A. Let n be this number. Then machine M starts to simulate the computation of the DFA A on all possible strings of length n to length $2n$. If DFA A accepts any such string then M accepts, othewise M rejects.

The key observation behind the above construction is that if there is a loop in the DFA, then there exists at least one string w with length $n \leq w \leq 2n$ that makes the DFA to loop.

Problem 2.

Problem 3.

Problem 4.

Problem 5.

Consider the problem of testing whether a Turing machine M on input w ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and show that it is undecidable.

Answer
The proof is by contadiction. Assume that this problem is decidable, then we will show that A_{TM} is also decidable. Let R be a Turing machine that decides the problem. That is, R decides the language $LEFTMOST = \{\langle M, w\rangle \mid M \text{ on input } w \text{ ever attempts to move its head left when it's hea d is on the left-most tape cell}\}$.

We can construct a Turing machine S that decides A_{TM} as follows. On input $\langle M, w\rangle$, S first modifies machine M to M', so that M' moves its head to the left from the left-most cell only when M accepts its input. To ensure that during its computation M' doesn't move the head left from the left-most position, machine M' first shifts the input w one position to the right, and places a special symbol on the left-most tape cell. The computation of M' starts with the head on the second tape cell. If during its computation M' ever attempts to move its head to the left-most tape cell, M' finds out that it did so by reading the special symbol and then puts the head back to the second cell, and continues its execution. If M would enter an accept state, then M' enters a loop that forces the head to move always to the left. After S has constructed M' it runs the decider R on input $\langle M', w\rangle$. If R accepts then S accepts, otherwise if R rejects then S rejects.