Homework 4

Due: Friday, Mar. 19, at 9:00pm in the handin bin located on 2^{nd} floor CIT

Collaboration is allowed only by terms of the policy. All problems are worth 20 points except the last one which is worthe 40 points. Tom will hold TA hours Thursday 8-9PM, Dave Friday 3-4PM in the TA annex room on the second floor of the CIT.

Problem 1.

Show that the PCP is decidable over the unary alphabet, that is, over the alphabet $\Sigma = \{1\}$.

Problem 2.

Show that the PCP is undecidable over the binary alphabet, that is, over the alphabet $\Sigma = \{0, 1\}$.

Problem 3.

Let T_c bet the set of Turing Machines with c or fewer states. The halting problem is trivally solvable for the set of machines T_1. Can the halting problem be solved for any other T_c with $c > 1$?

Problem 4.

Define a ***two headed finite automaton*** (2DFA) to be deterministic finite automoton that has two read-only, bidirectional heads that start at the left-hand end of the input tape and can be independently controlled to move in either direction. The tape of a 2DFA is finite and is just large enough to contain the input plus two additional blank tape cells, one on the left-hand end and one on the right-hand end, that serve as delimeters. A 2DFA accepts its input by entering a special halt state. For example, a 2DFA can recognize the language $\{a^n b^n c^n | n \geq 0\}$.

a. Let $A_{2DFA} = \{\langle M, x\rangle | M \text{ is a 2DFA and } M \text{ accepts } x\}$. Show that A_{2DFA} is decidable.
b. Let $E_{2DFA} = \{\langle M\rangle | M \text{ is a 2DFA and } L(M) = \emptyset\}$. Show that E_{2DFA} is not decidable.