Homework 7

Due: Friday, Apr. 30, at 9:00pm in the handin bin located on 2^{nd} floor CIT

Collaboration is allowed only by terms of the policy. All problems worth 20 points. Tom will hold TA hours Thursday 8-9PM, Dave Friday 3-4PM in the TA annex room on the second floor of the CIT.

Problem 1.

Show that, if every NP-hard language is also PSPACE-hard, then PSPACE = NP.

Problem 2.

Let A be the language of properly nested parentheses. For example, $(())$ and $(()(()))()$ are in A, but $)($ is not. Show that A is in L.

Problem 3.

Recall that a directed graph is *strongly connected* if every two nodes are connected by a directed path in each direction. Let

$$STRONGLY-CONNECTED = \{\langle G\rangle | G \text{ is a strongly connected graph}\}.$$

Show that $STRONGLY-CONNECTED$ is NL-complete.

Problem 4.

Show the error in the following fallacious "proof" that $P \neq NP$. Proof by contradiction. Assume that $P = NP$. Then $SAT \in P$. So, for some k, $SAT \in \text{TIME}(n^k)$. Because every language in NP is polynomial time reducible to SAT, $NP \subseteq \text{TIME}(n^k)$. Therefore $P \subseteq \text{TIME}(n^k)$. But, by the time hierarchy theorem, $\text{TIME}(n^{k+1})$ contains a language which isn't in $\text{TIME}(n^k)$, which contradicts $P \subseteq \text{TIME}(n^k)$. Therefore $P \neq NP$.

Problem 5.

Sipser 9.18