Homework 1

CS157 - Spring 2009

Due: Tuesday February 3, 2008 10:30am

Problem 1

Show that

$$\log(n!) = \Theta(n \log n)$$

Hint: To show an upper bound, compare $n!$ with n^n. To show a lower bound, compare it with $(n/2)^{n/2}$

Problem 2

Provide solutions for the following recurrence relations using general techniques, but do not use the Master Theorem. We ask that you give a proper proof for a recurrence relation of your choice, but you can just provide answers for the other three.

If your recurrence relation involves a non-integral value (e.g. $n/2$ for n odd), just suppose that you'll use the integral part.

Do not worry about lower-order terms or constant factors: we want an answer $f(n)$ which satisfies $T(n) = \Theta(f(n))$.

- $T(n) = T(\frac{2n}{3}) + 1, \qquad T(0) = T(1) = 1$
- $T(n) = 3T(n-5) + n, \qquad T(n)$ is O(1) when $n \leq 5$
- $T(n) = 2T(n/2) + n^2, \qquad T(0) = T(1) = 1$
- $T(n) = T(\lceil n/5 \rceil) + T(\lceil 7n/10 + 6 \rceil) + n, \qquad$ T(n) is O(1) when $n \leq 23$

Problem 3

Rank the following functions by order of growth; that is, find an arrangement $g_1, g_2, \ldots, g_{10}$ of the functions satisfying $g_9 = o(g_{10}), g_8 = o(g_9)$, etc. You need not justify your answer.

$$7n^3 + 3n \ , 4n^2 \ , n \ , \log_{\sqrt{n}}(n^6) \ , \frac{1}{n^6} + 18n^5 \ , n^{8621909} \ , 3^n \ , e^{\log \log n} \ , 2^{3^n} \ , n^{\log n} \ , 6n \log n \ , n!.$$

Problem 4

For this problem consider the implementation of the function `merge` on page 51 of the textbook. For input $x[1, ...k]$ and $y[1, ...l]$ where $k = l = 4$ give two input arrays that are the best case for merge and two input arrays that are the worst case for merge.