Homework 2

CS157 - Spring 2009

Due: Tuesday February 17, 2008 10:30am

Problem 1: Fourier Transform in modular arithmetic

This problem illustrates how to do the Fourier Transform (FT) in arithmetic modulo a prime, for example modulo 5. *Note:* $p = 0 \pmod{p}$ *but* $y^p \neq y^0 \pmod{p}$.

(a) Multiply the polynomials $3x^2 + x + 2$ and $2x + 1$ by hand (using the naïve algorithm), doing all arithmetic modulo 5.

(b) There is a number ω such that all the powers $\omega, \omega^2, \omega^3, \omega^4$ are distinct (modulo 5). Find all such ω. For the remainder of this problem let ω be the largest of these ωs, where by "largest" we mean $0 < 1 < 2 < 3 < 4$. (Interestingly, for any prime modulus there is such a number.)

(c) Show that $\omega + \omega^2 + \omega^3 + \omega^4 = 0 \pmod{5}$.

(d) Write down the Fourier Transform matrix $M_4(\omega)$ (substituting the numerical value from part (b) for ω). All calculations should be performed modulo 5.

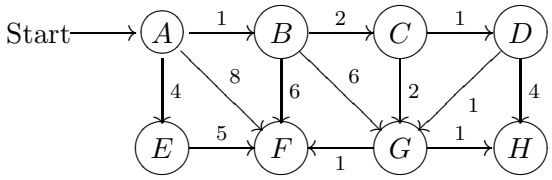
(e) Produce the Fourier Transform of the vector $(2, 1, 3, 0)^T$ modulo 5.

(f) Write down the matrix necessary to perform the inverse FT. Show that multiplying the answer to the previous part by this matrix returns to the original sequence.

(g) Perform the polynomial multiplication of part (a) again using the FT. Check that you get the same answer as in part (a)!

Problem 2: Dijkstra

Suppose Dijkstra's algorithm is run on the following graph, starting at node A.



(a) Draw a table showing the intermediate distance values of all the nodes at each iteration of the algorithm.

(b) Show the final shortest-path tree.