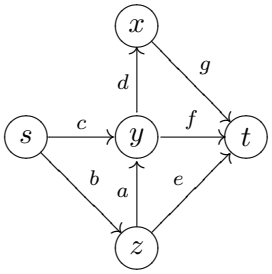
Homework 5

CS157 - Spring 2009

Due: Tuesday April 14, 2009 10:30am

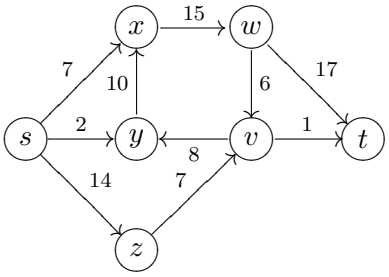
Problem 1: Graph Cuts

Here is a network with vertices $V = \{s, t, x, y, z\}$ and edge capacities $\{a, b, c, d, e, f, g\}$. List all (s, t)-cuts and their respective capacities, with the terms in each sum given in alphabetical order. (Recall that an (s, t)-cut is a partition of V into disjoint sets S and T such that $S \cup T = V$ and $s \in S$, $t \in T$. The capacity of such a cut is the total capacity of all edges from S to T.)



Problem 2: Max-Flow/Min-Cut

(a) Give a maximum flow from s to t on the following network. What is the value of this flow?



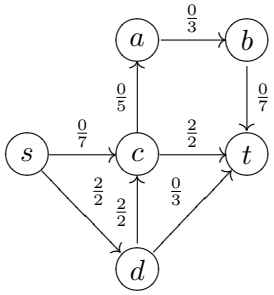
(b) Give a minimum (s, t)-cut. What is the value of this cut?

(c) Explain *briefly* why we can be sure the values from the previous two parts are maximum and minimum respectively. (*Hint*: Take a look at the title of this question.)

Problem 3: The Edmonds-Karp Algorithm

(a) The following graph shows an s-t flow, where the edge labels are $\frac{\text{flow}}{\text{capacity}}$. Give the residual graph.

(b) Give the augmenting path that would be selected by the Edmonds-Karp Algorithm.



(c) Give the resulting flow after the augmenting path has been added.

Problem 4: Linear Programming

Write the following linear program as a maximization over four variables in standard form:

$$\min x_1 + 3x_2 - 2x_3 + 6x_4$$

subject to

$$\begin{aligned} 4x_2 - 8x_3 &\leq 400 + 2x_1 \\ 25 - 6x_1 &\geq 3x_2 + 4x_4 \\ x_3 &\leq 7x_1 + 42 \\ x_1 &\leq -2 \\ x_2 &\geq 4 \\ x_3 &\geq 1 \\ x_4 &\leq 0 \end{aligned}$$