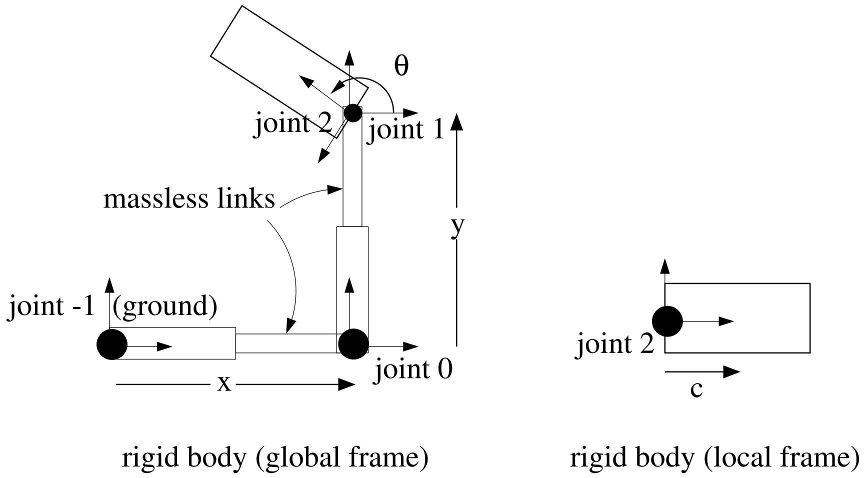

Figure 1: Joint locations and position parameters for a single rigid body.

Homework 3: Spatial Vectors

Suppose you want to use Featherstone's formulation to implement an inverse kinematics routine for a single rigid body. The body is modeled as having a massless prismatic joint for motion in the x-direction, followed by a massless prismatic joint for motion in the y-direction, followed by a rotational joint located at one end of the cylindrical body. Figure 1 shows the rigid body situated in the global frame and also situated in its local frame (the joint 2 coordinate frame). Vector c runs from the origin of the joint 2 frame to the center of mass of the body.

1. What parameters make up the generalized position ($\mathbf{q}$) and generalized force ($\mathbf{Q}$) vectors for the rigid body?
2. What are the local frame joint axis vectors $\hat{s}_i^{'}$?
3. Please give values for all of the transformation matrices ${}_i\hat{X}_{i+1}$.
4. What are the local coordinate frame inertia matrices $\hat{I}_i^{'}$?
5. How would you deal with gravity? (Be specific, using parameters that are part of the inverse dynamics formulation.)
6. How would you add a torsional spring at the object center of mass that was designed to keep the body oriented in the vertical (y) direction? (Be specific, using parameters that are part of the inverse dynamics formulation.)
7. (BONUS) In the inverse dynamics example for a particle, results are provided for all of the spatial force vectors $\hat{f}_i$ (Spatial Vectors Handout, Section 3). The terms in the upper half of $\hat{f}_0$ and $\hat{f}_{-1}$ are the familiar mass-times-acceleration terms from Newton's second law of motion. What is the meaning of the terms in the lower half of $\hat{f}_0$ and $\hat{f}_{-1}$?