CS250B Homework 1
Due Thursday, February 12, 2009, at 2:30 pm

1. Construct a (combinatorially) embedded graph that is not planar. Use the minimum number of nodes and the minimum number of edges.

2. Prove that there is a constant c_1 such that every n-node-connected planar embedded graph with no parallel edges or self-loops has at most $c_1 n$ edges.

3. Write a program for the following:

   - input: an embedded graph T that is a tree, and a dart d_0.
   - output: the set of darts d such that d lies on a simple path in T starting with d_0.

   You can use Python, c, c++, Java, or Scheme. Check with me if there is another language you wold prefer.

4. Prove that there are positive constants c_2, c_3 such that an n-node planar graph with no parallel edges has at least $c_2 n$ vertices of degree c_3 or less.

5. Prove the following theorem:

   > Let G be a planar embedding graph. If T is a spanning tree of G then the edges $E(G) - E(T)$ form a spanning tree of G^*.

   You may use results proved in the lecture notes before the statement of this theorem.

6. Give a linear-time algorithm for the following:

   - input: a planar embedded graph G, and a spanning tree T of G
   - output: a table giving, for each nontree edge uv, the length of the simple u-to-v path in T between u and v.

7. Give a linear-time algorithm for minimum-weight spanning forest tree in a planar graph. You are encouraged to use the following results. You need not prove the results.

   (a) Let G be a graph with edge-weights, and let v be a vertex. Let e be the minimum-weight edge incident to v. Then G has a minimum-weight spanning tree that includes e.

   (b) Let G be a graph with edge-weights, and let e be an edge contained by some minimum-weight spanning tree of G. Let T be a minimu-weight spanning tree of T/e. Then $T \cup \{e\}$ is a minimum spanning tree of G.