CS250B Homework 5

Due Tuesday, May 5, 2009, at 2:30 pm

1 Elementary cycle separators

Give a counterexample disproving the following "lemma":

"lemma" *Given a spanning tree T of a planar graph, given face-weights such that every face weight is at most 1/3 of the total, there exists an elementary cycle that is a 1/3-2/3 separator.*

2 Miller's algorithm for $O(\sqrt{n})$ simple cycle separator

Part of Miller's algorithm involves constructing a rooted auxiliary tree $\mathcal{T}$ based on breadth-first search of the dual G^*, as described in Lecture 16. Give a linear-time algorithm to construct $\mathcal{T}$ from G^* and a breadth-first search of G^*.

3 Miller, Rose, and Tarjan's algorithm for shortest paths

Show that Miller, Rose, and Tarjan's algorithm for shortest paths in planar graphs uses $O(n \log n)$ space.

4 Vertex-cover in low-branch-width graphs

Give the algorithm that proves this theorem:

Theorem: *There is an algorithm that, given a graph G with edge-weights and a branch-decomposition $\mathcal{C}$ of G, finds the minimum-weight vertex-cover of G in time $O(2^w n)$ where w is the width of $\mathcal{C}$.*

5 Baker's technique: edge cover

Describe how to apply Baker's technique to give an approximation scheme for weighted edge-cover in planar graphs.

6 Baker's technique: max-weight independent set

Describe how to apply Baker's technique to give an approximation scheme for max-weight independent set.

7 Lipton and Tarjan's algorithm for $O(\sqrt{n})$ vertex separators; recursive decomposition

Give a linear-time algorithm to construct a recursive decomposition based on the $O(\sqrt{n})$ vertex separators of Lipton and Tarjan.