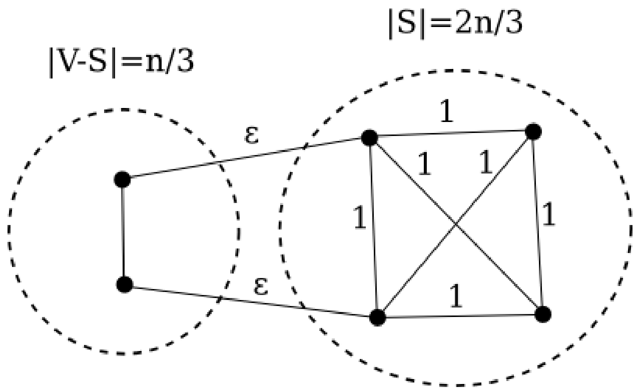
Problem Set 10

Problem 21.3

To show that the psuedo-approximation algorithm for minimum (1/3)-balanced cut cannot easily be converted to a true approximation algorithm, we give a class of instances on which the minimum (1/3)-balanced cut is cheaper than the minimum bisection by an arbitrarily large factor.



The instance consists of a complete graph $G = (E, V)$ (some edges are omitted from the diagram for clarity) and non-negative cost function $c : E \to \mathbb{R}^+$. For some fixed set $S \subset V$ with $|S| = 2n/3$, the cost function is defined as

$$c(v_1, v_2) = \begin{cases} \varepsilon & \text{if } (v_1, v_2) \in S \times (V - S) \\ 1 & \text{otherwise} \end{cases}$$

Since any bisection must contain at least one edge in $S \times S$, it must have cost at least 1. The cut $(S, V - S)$ is a (1/3)-balanced cut with cost $\varepsilon \cdot |S \times (V - S)|$, which can be made arbitrarily small.