Approximation Algorithms
Homework #11

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Exercise 22.4

Consider graph G in Figure 1 where $S_1 = \{u, v_1\}$, $S_2 = \{u, v_2\}$ and $S_3 = \{u, v_3\}$ and edge costs are 2.

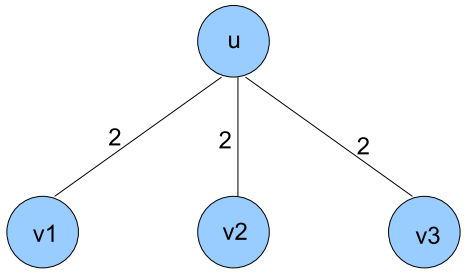

Figure 1: Graph G.

Let is track how Algorithm 22.3 performs on the linear programming formulation of Steiner Forest problem on graph G.

Initialization: There are four active singleton sets $\{u\}$, $\{v_1\}$, $\{v_2\}$ and $\{v_3\}$. Consider set $C = \{u\}$ and cut $(C, \bar{C})$ raised by the algorithm. Note that $\delta(C) = \{u\text{-}v_1, u\text{-}v_2, u\text{-}v_3\}$.

Edge augmentation: After the first step all edges become tight. Since there are no more active sets algorithm terminates with $F = \{u\text{-}v_1, u\text{-}v_2, u\text{-}v_3\}$

Pruning: Neither of the edges can be removed from F since primal solution will be infeasible with the removal of either of these edges. Hence $F' = F$.

Now consider $deg_{F'}(C)$ - the number of the edges from primal solution F' that are present in the cut C. $deg_{F'}(C) = |F' \cap \delta(C)| = 3$.