

CSCI 2510 - Problem set 10

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Analysis of the greedy online bipartite matching algorithm

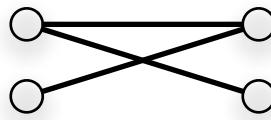
Given an online bipartite matching problem $G = (U, V, E)$, the greedy online algorithm referred to in class chooses to match a node u as it is revealed appears to the first (in some deterministic canonical order) non-matched v if such v exists. This algorithm is 2-competitive. to see that consider the optimal offline matching OPT .

Lemma 1. *for every edge (u, v) in OPT , at least one of u, v is matched by the greedy algorithm.*

Proof. Consider the time u is revealed. If u is matched by the greedy algorithm to some node in V we are done. Otherwise, all nodes in V that connect to u were already matched by the greedy algorithm. In particular v was matched by the greedy algorithm. \square

It follows that the size of the matching produced by the greedy algorithm is at least half the size of the matching produced by OPT , hence the algorithm is 2-competitive.

As we saw in class, this analysis is tight. Consider a graph that consists of n copies of the gadget shown in the figure, where the canonical order simply lists the vertices from top to bottom. The greedy algorithm matches the first two nodes and cannot match the remaining two. The



optimal solution can match all vertices. Thus the ratio of the matchings is $\frac{2n}{n} = 2$.