Approximation Algorithms
Homework 2
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Class problem

Decide whether the algorithm discussed in class (described below) is a $2(1-\frac{1}{k})$ *approximation.*

Algorithm 1

$E' \Leftarrow \{\}$
for $i = 1$ to k **do**
  $E' \Leftarrow E' \bigcup mincut(S_i, \{S_{i+1}, S_{i+2}, ..., S_k\})$
end for
return E'

Theorem 1. *Algorithm 1 does not produce a* $2(1-\frac{1}{k})$ *approximation to the multiway cut problem.*

Proof. The Figure below shows an example that produces an approximation that is greater than the necessary $2(1-\frac{1}{k})$.

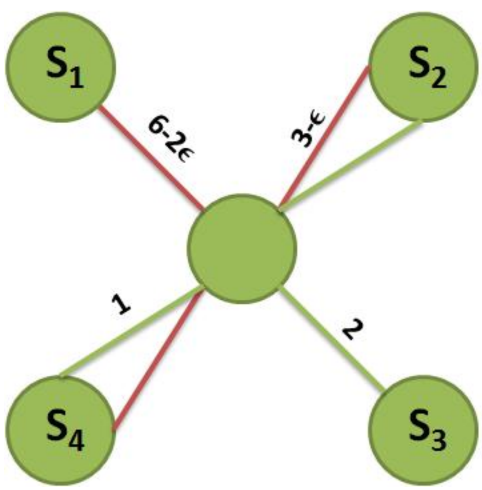

Figure 1: Red edges are those included by the algorithm, green edges are those in the optimal cut.

As can be seen in this diagram, the optimal cut has a cost of $6 - \epsilon$ while because of the particular ordering of the required nodes the cost of the algorithm picked edges is $10 - 3\epsilon$. This is of course greater than the required $9 - \frac{3}{2}\epsilon$ for the $2(1 - \frac{1}{k})$ approximation.

This algorithm almost passes the required approximation because it ignores the last cut between S_{k-1} and S_k, however because the nodes have an arbitrary ordering, there is no garauntee that the removed cut will be the most expensive cut.

$\square$