Approximation Algorithms
Homework #5

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Simplex Exercise 1

$$\begin{aligned} \text{maximize} \quad & 32 + 10x_4 \\ & x_1 + 2x_4 = 8 \\ & x_2 + 3x_4 = 6 \\ & x_3 + 8x_4 = 24 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Part (a)

$$\begin{aligned} \text{maximize} \quad & 32 + 10x_4 \\ & x_1 = 8 - 2x_4 \\ & x_2 = 6 - 3x_4 \\ & x_3 = 24 - 8x_4 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

The problem is given in the canonical form and thus $x_1 = 8$, $x_2 = 6$, $x_3 = 24$, $x_4 = 0$ is a feasible solution with the objective value of 32. However objective function still contains x_4 with a positive coefficient and thus we can improve the value of the objective function by increasing x_4. x_4 is our pivot variable (entering variable). We pick second constraint to solve for x_4 as x_4 is most constrained by it

$$x_4 = 2 - x_2/3 \tag{1}$$

Variable x_2 is a leaving variable, i.e. new non-basic variable. We substitute x_4 in all constraints and the objective function with relationship in Equation (1). The system becomes

$$\begin{aligned} \text{maximize} \quad & 52 - 10x_2/3 \\ & x_1 = 4 + 2x_2/3 \\ & x_4 = 2 - x_2/3 \\ & x_3 = 8 + 8x_2/3 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Since x_2 is the only non-basic variable and appears with a negative coefficient we cannot improve our objective value. Hence, 52 is an optimal solution with $x_1 = 4$, $x_2 = 0$, $x_3 = 8$ and $x_4 = 2$.

Part (b) With coefficient of x_4 changed to -3 in the objective function we obtain

$$\begin{aligned}\text{maximize} \quad & 32 - 3x_4 \\ & x_1 = 8 - 2x_4 \\ & x_2 = 6 - 3x_4 \\ & x_3 = 24 - 8x_4 \\ & x_1, x_2, x_3, x_4 \geq 0\end{aligned}$$

Basic feasible solution to this problem is optimal solution as we cannot increase x_4 to improve the objective value of 32. Hence, 32 is an optimal solution with $x_1 = 8$, $x_2 = 6$, $x_3 = 24$ and $x_4 = 0$.

Part (c) By changing the sign of x_4 coefficients in all constraints we obtain

$$\begin{aligned}\text{maximize} \quad & 32 + 10x_4 \\ & x_1 = 8 + 2x_4 \\ & x_2 = 6 + 3x_4 \\ & x_3 = 24 + 8x_4 \\ & x_1, x_2, x_3, x_4 \geq 0\end{aligned}$$

Note that constraints in the above problem no longer constrain the value of x_4: increasing of x_4 does not decrease none of the basic variables x_1, x_2 and x_3. Hence, x_4 can be increased forever meaning that the objective function is unbounded.