Approx. Algo. HW7

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1 Problem 16.1

Here we will give a tight example of factor $1 - \frac{1}{2^k}$ for the algorithm in section 16.1.

Assume we have k variables, we can enumerate 2^k different clauses of size k, by using the binary representation. Suppose the clauses are $C_0, ..., C_{2^k-1}$, C_i is defined as

$$\bigvee_{\text{j-th digit of i is 1}} X_j \bigvee_{\text{j-th digit of i is 0}} \neg X_j$$

where i is represented as an binary number.

Since we have enumerate all the possible outcomes of the variable assignment, there is always one clauses unsatisfied, which is the clause with literals that are the negation of the corresponding assignment.

The weight of C_0 is 0 and all the rest clauses $(C_1, .., C_{2^k-1})$ are with weight $\alpha > 0$.

The optimal solution would be assigning all variable to $true$, hence C_0 is unsatisfied. The optimal value is $(2^k - 1)\alpha$.

The value of any solution other than the optimal is $(2^k - 2)\alpha$.

Hence the expected value is $\frac{1}{2^k}(2^k - 1)\alpha + \frac{2^k-1}{2^k}(2^k - 2)\alpha = \frac{(2^k-1)^2}{2^k}\alpha$

Hence the approximation factor is the expect value of the approx. algo. over the opt. value,

$$\frac{\frac{(2^k-1)^2}{2^k}\alpha}{(2^k - 2)\alpha} = 1 - \frac{1}{2^k} = \alpha_k$$

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