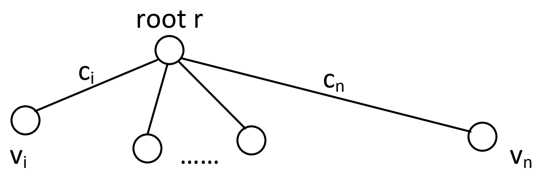
CS251 – Hw 8 Serdar Kadioglu
(serdark)

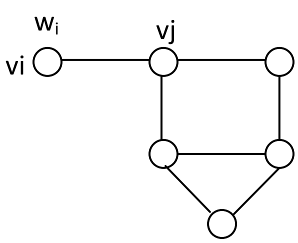
HOMEWORK - 8

18.1 Suppose, we are given an instance of a multicut problem on a tree T of height 1 with root *r*, vertices $v_1, \ldots, v_n$ and edges $ei = (r, v_i)$ with cost c_i.



First we remove edges $e_i = (r,v_i)$ if r, v_i is a multicut pair. Then, create a graph H whose vertices are the pairs in the given multicut instance and whose has an edge for each such pair. Now, consider the (weighted) vertex cover problem on H where the weight of a vertex v_i is c_i. If edges $e_i,\ldots,e_k$ forms a multicat in T, then the vertices $v_i, \ldots, v_k$ form a vertex cover in the graph H, and vice versa. Therefore, finding the minimum multicut in T is equivalent finding a minimum weight vertex cover in H. Also, vertex cover instance inherited the same weights for its vertices from our original edge weights, preserving the approximation factor.

Analogously, given an instance of vertex cover C, we can construct a tree T of height 1 that with all vertices in C are connected to a dummy root with edge costs are being vertex weights. And for each edge $e = (a,b)$ in C we have a pair (a,b) in the multicut problem.



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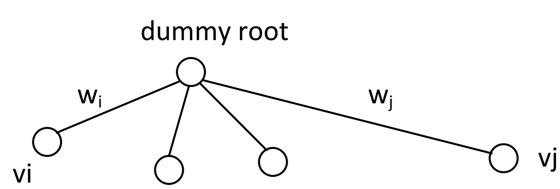

Vertex Cover Instance
vertices v_i weights w_i

MultiCut Instance
vertices v_i and *r*, edges e_i=w_i, pairs(v_i. v_j)

If $v_x \ldots v_y$ is a (min) vertex cover then, $e_x \ldots e_y$ is a (min) multicut