Approximation Algorithms

Homework #10

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Exercise 20.7

Graph bipartization by edge deletion

Input: Weighted undirected graph $G = (V, E)$.

Output: Minimum weight set of edges E' such that $G' = (V, E - E')$ is bipartite.

We show a polynomial time reduction of the above problem to 2CNF≡ clause deletion problem.

Reduction: We construct a 2CNF≡ formula F as follows. For each node $v \in V$ we introduce a Boolean variable b_v, such that if b_v is *true* then v is in component 1, otherwise in component 2. Since we want each node to be connected only to nodes from a different component, for each edge v-u we add a clause $(b_v \equiv \bar{b}_u)$ to F. Each clause in F is assigned to a weight equal to the weight of a corresponding edge. Note that F is satisfiable iff adjacent vertices are in different components.

Consider a solution to 2CNF≡ clause deletion problem on F: a minimum weight set of clauses C that needs to be removed from F to make F satisfiable. As each clause in C corresponds to an edge in graph G we obtain $E' = \{u\text{-}v \mid \forall (b_u \equiv b_v) \in C\}$. $G' = (V, E - E')$ is bipartite since any satisfiable assignment for $F - C$ corresponds to a partition of V in G' where no two adjacent vertices are in the same component.

Since we have an $O(\log n)$ approximation algorithm (Theorem 20.14) for a 2CNF≡ clause deletion problem, we have an $O(\log n)$ approximation algorithm for a graph bipartization by edge deletion problem. □