

Homework 2

Problem 4.7

Prove that if the Gomory-Hu tree for an edge-weighted undirected graph G contains all $n-1$ distinct weights, then G can have only one minimum weight cut.

Proof: Let G be the graph $G = (V, E)$. Let T be the corresponding Gomory-Hu tree. The edges of T represent the minimum cost partition of V into two sets S, \bar{S} such that the vertices corresponding to the nodes on one side of the edge are in S and the vertices corresponding to the nodes of the other side are in \bar{S} . Thus, for each pair of vertices (u, v) , T contains all minimum weight partitions of u from v . Therefore the Gomory-Hu tree contains all minimum cuts. Therefore if the edges on the Gomory-Hu tree are of distinct weights then there is only one minimum weight cut, the cut that corresponds to the minimum weight edge in T .